How Aging Laws Influence Parametric and Catastrophic Reliability Distributions

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SUMMARY & CONCLUSIONS

In this paper we describe how these physics of failure aging laws influence reliability distributions, not only the type of distribution, but the rate of failure as it relates to the aging rate. We illustrate how one can predict parametric failure rates based on the physics of failure aging laws when known.

A number of statements are concluded. We show that when a manufactured part has a key parameter that is distributed normally, and the physics of failure aging for this key parameter ages in log-time, its failure rate is lognormally distributed.

When the physics of failure is a power law, we illustrate how the Weibull beta and eta can be obtained from the physics of failure aging law exponent and amplitude in the parametric case. We use the example of creep, and make direct comparisons between the full creep 'rate' curve and the bathtub curve. Although the example of creep is used many aging laws have a similar power law forms and can be applied in a similar manner. Although we work though parametric failure rate statistics, one can relate it to the catastrophic case.

1 INTRODUCTION

The most popular aging laws that are observed in the literature are:

Power Law: $P(t) = Ct^{K}$
where P is the aging parameter of interest, C and K are constants, and t is time.

Log time Aging: $P(t) = A\ \ln(1+bt)$
where A, and b are constants, P is the parameter of interest and t is time.

Arrhenius Activation: $R = R_o\ e^{-E_a/K_BT}$
where R is the aging rate of a parameter, R_o is a constant related to a time constant $1/t_o$. The rate R goes inversely with time.

It is common if the aging law is known to simply plug it into the distribution dependence. Then the rate of failure becomes known by fitting failure data. However, when parameters are unknown, what can we say about the rate of degradation from the observed failure rate statistics. Alternately, if the physics of failure law is know, what can we say about the parametric failure rate. There are two main types of failure processes that are typically analyzed in reliability. These are catastrophic and parametric failure rates. Often parametric failure is analyzed using catastrophic analysis, a pass fail approach using a parametric threshold. For example, greater than 15% change might be considered a threshold for a failure. Although, when we look a little closer, using parametric failure statistics instead, other insights can be found. Such influences in parametric data can in fact be related to many seemingly catastrophic failure events. That is, what appears as a catastrophic event is often due to an underlying parametric aging process that eventually abruptly fails. In this case, most of the degradation lifetime prior to catastrophic abrupt failure, actually occurred due to a graceful aging that can be modeled. As an example, creep abruptly fails at end of life, yet most of its lifetime can be modeled prior to catastrophic failure with a graceful parametric aging law using either a power or a log time aging law. In this paper we provide a number of examples using both Weibull and lognomal statistics that demonstrate how we can associate the reliability statistics to aging model parameters.

Reliability distributions actually are designed to fit regions of the bathtub curve shown in Figure 1. The bathtub curve is shown below.

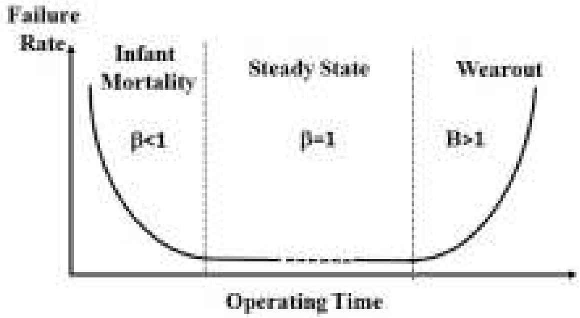

Figure 1 Reliability bathtub curve model

For example, the Weibull distribution is basically a power law over time. If we were to invent a distribution based on wear-out for example, we might use a failure rate λ(t) proportional to time raised to a power greater than one. For example, wear-out on a particular device may fit a power law with time squared as

$$\lambda = \lambda_o t^2 \tag{1}$$

This is essentially a Weibull failure rate, although the actual Weibull failure rate is written in a more sophisticated way as

$$\lambda(t) = \frac{\beta}{\alpha^\beta}(t)^{\beta-1} \tag{2}$$

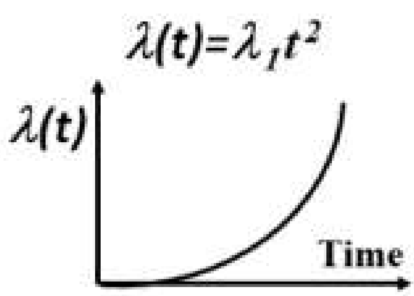

Figure 2 Power law fit to the Wearout portion of the bathtub curve.

This is still a power law and in this case, beta would equate to 3. Note alpha is the characteristic life. Many reliability engineers tend to favor the Weibull distribution because of the physical significance of the beta parameter. That is the power exponent in the distribution helps to determine the portion of the bathtub curve that we are in. Often semiconductor engineers work with the lognormal distribution [1,2].

True statisticians likely will tell you whichever distribution fits the data best; will provide the most accurate assessment. However, we might ask a deeper question, what is the physics of failure law influencing the failure rate distribution. How does the statistical distribution help us to determine the physics of failure models or alternately how does the physics of failure aging rate help determine the parametric failure rate statistics? In this paper we will examine the connection between the physics of failure aging laws and the parametric failure rate distribution statistics. Therefore, our initial discussion will revolve around parametric reliability distributions. Once these are established, we can often infer how the catastrophic distribution will follow or alternately we can help from failure rate data determine physics of failure models.

2 LOG TIME AGING AND THE LOGNORMAL DISTRIBUTION

We know from production, that many parameters tend to be normally distributed. For example, the following parameters were found to be normally distributed, quartz crystal frequencies [6->3], beta gain for transistors [8->4], transconductance of FET semiconductors [4]. We have selected these in particular as the physics of failure parametric aging law is in log time. That is:

> ***We would like to illustrate that if parts are normally distributed and age in log-time, then their failure rate is lognormal. Furthermore, we can demonstrate that power laws (like that shown in Eq. 3) where the aging exponent for time (K is between zero and one, can be also modeled as aging in log-time, this means that the true physics may actually not be a power law but instead, a log time law.***

To that end, the general form of the a log time model for these devices is found to be [3, 4]

$$P = A \ln(1 + bt) \approx Ct^K \quad \text{where } 0<K<1 \tag{3}$$

and to simplify, when bt>>1 we can write

$$P \approx A \ln(bt) \tag{4}$$

Here P is the parameter of interest, such as beta transistor aging, or crystal frequency drift and so forth.

In order to have parametric failure, one needs a definition for failure. To this end it is customary to define a parametric failure threshold. That is, when a component ages, one of its key parameters drifts out of specification. This value can be used as the failure threshold. For example, transistor beta degradation can be taken as a change of 10 or 20% of the original value. The figure below depicts how these key parameter age in log time failure. In the figure the threshold is given at P=1.37 for time t=3. Here time units are not defined but are usually in hours or months and so forth.

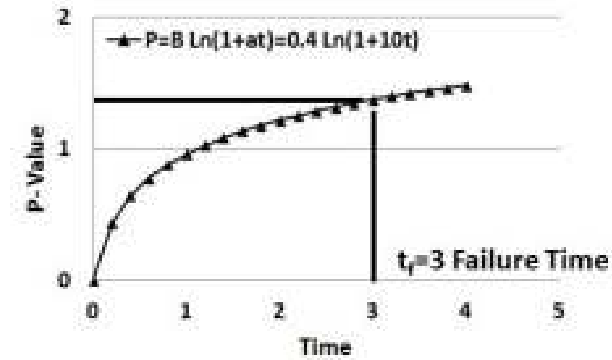

Figure 3 Log time aging with parametric threshold t_f

When manufactured parts are normally distributed, a parameter of interest can be statistically assessed using Gaussian probability density function (pdf), *g(p, t)*

$$g(p,t) = \frac{1}{\sigma\sqrt{2\pi}} \quad \exp\left[-\frac{1}{2}\left(\frac{p(t)-\bar{p}(t)}{\sigma}\right)^2\right] \tag{5}$$

Here *P* is the parameter of interest. Now consider that the parameter is aging according to a log-time equation such as Equation 3, its time dependence must then be lognormally distributed, that is, we have from Equations 4 and 5

$$g(\ln t : t_{50},\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \quad \exp\left[-\frac{1}{2}\left(\frac{\ln t-\ln t_{50}}{\sigma}\right)^2\right] \tag{6}$$

Where for purposes of illustration in Equation 4 we have let A=b=1

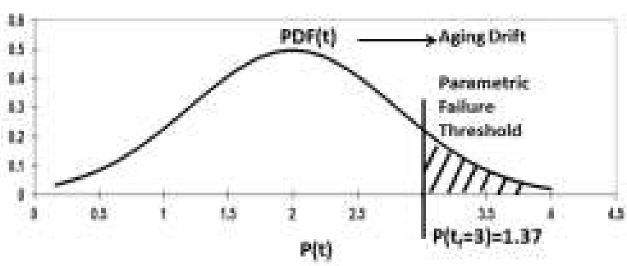

Figure 4 PDF failure portion that drifted past the parametric threshold

It is customary to change variables so that we may formally obtain the lognormal distribution for the above equation, then

$$g(\ln t)d\ln t = g(\ln t)\frac{d\ln t}{dt}dt = \\ g(\ln t)\frac{dt}{t} = \ln g(t)dt \tag{7}$$

We can now write with this change of variables

$$\ln g(t : t_{50},\sigma) = f(t : t_{50},\sigma) \\ = \frac{1}{\sigma t\sqrt{2\pi}} \quad \exp\left[-\frac{1}{2}\left(\frac{\ln t-\ln t_{50}}{\sigma}\right)^2\right] \tag{8}$$

Here, the function $f(t{:}t_{50},\ \sigma)$ is the lognormal probability density function.

The cumulative distribution function *cdf* , F(t) is related as

$$f(t) = \frac{dF(t)}{dt} \tag{9}$$

The cdf for the lognormal distribution can be written in closed form with the help of the error function (*erf*) as

$$F(t) = \frac{1}{2}\left[1+erf\left(\frac{\ln t-\ln t_{50}}{\sqrt{2}\sigma}\right)\right] \tag{10}$$

Often, one writes the lognormal mean as

$$\mu = \ln(t_{50}) \tag{11}$$

And the dispersion is assessed graphically as

$$\sigma = \ln(t_{50}/t_{16}) \tag{12}$$

Thus, the physical implications can be related to log-time aging, in detail,

> ***when a manufactured part has a key parameter that is distributed normally, and ages in log-time, its failure rate is generally lognormally distributed. This is likely the case for power law aging models that are typically found empirically as well when the power 0<K<1 in Equation 3.***

Although we have described this for parametric failure, it can be argued that many catastrophic failure mechanisms dominated by log-time aging will also fall into this category. For example, if a transistor is aging most of its lifetime in log-time then suddenly fails catastrophically, but it was due to the underling log-time aging mechanism like gate leakage, then the transistor's failure distribution is likely lognormal. The parametric threshold in this case resulted in a true catastrophic failure event with most of its lifetime aging logarithmically in time.

We exemplify with a common log time aging model [2,5,6, 7], writing an aging parameter *P* with log-time aging form

$$\overline{P} = \overline{A}\left\langle \ln(1+bt)\right\rangle_{ave} \cong \overline{c} + \overline{A}\ln\left(t_{50}\right) \tag{13}$$

where the approximation is for bt>>1. Here $\overline{c}, \overline{A}$ are average values and $\overline{c} = \overline{A}\left\langle \ln b\right\rangle_{ave}$ and $\ln(t_{50})$ is the mean of [ln(t)] failure time. Then Equation 8 for the parametric PDF becomes

$$f(t:t_{50},\sigma) = \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\{c + A\ln(t)\} - \{\bar{c} + \bar{A}\ln(t_{50})\}}{\sigma}\right)^2\right] \qquad (14)$$

The CDF is then

$$F(t) = \frac{1}{2}\left[1 + erf\left(\frac{\{c + A\ln(t)\} - \{\bar{c} + \bar{A}\ln(t_{50})\}}{\sqrt{2}\sigma}\right)\right] \qquad (15)$$

2.1 What Have We Shown?

If we have a bunch of devices with a key parameter that we know obeys Eq. 13., we can now, prior to any aging, find the mean and sigma using simple Gaussian statistics measuring typically 30 non aged devices. Now we can take from this population a few devices, say 3 devices and perform an aging experiment on them, we then find the parameters of the physics of failure aging law in Equation 13. We now immediately know the lognormal failure rate statistics without having to do a full life test on 1) a large population and 2) we never actually have to age any of the 3 devices to failure! We can establish a failure threshold and predict when 50% of the population will pass this threshold (see Figure 4). Alternately, if we find the parametric lognormal failure rate, we can determine the parameters of the physics of failure aging law in Eq. 13.

3 AGING POWER LAWS AND THE WEIBULL DISTRIBUTION – INFLUENCE ON BETA

Many parametric aging laws have power law dependence. Consider creep as an example

$$\Delta\varepsilon = at^n \qquad (16)$$

where $\Delta\varepsilon$ is the creep strain and t is the time, and a and n are constants of the creep model [8]. This simple equation can actually model both the primary and secondary creep phases [8], but not the 3rd stage tertiary creep phase as shown in Figure 5.

Now we would like to provide some new understanding to the Weibull distribution and how underlying aging laws might influence the distribution or how analysis might help us in determining an aging law. As a point of reference, we write the popular Weibull failure rate as

$$\lambda(t) = \frac{\beta}{\alpha^{\beta}}(t)^{\beta-1} \qquad (17)$$

For the traditional Weibull model, $\beta<1$ is infant mortality, $\beta=1$ is steady state, and $\beta>1$ is wear-out.

There are traditional functions to help obtain the failure rate in reliability statistics. The functional definition for the instantaneous failure rate are defined with

$$\lambda(t) = -\frac{d\ln R(t)}{dt} = \frac{f(t)}{R(t)} = \frac{f(t)}{1-F(t)} \qquad (18)$$

Where R(t) is the reliability function, f(t) is the pdf, and F(t) is the CDF.

However, for what we wish to do we are going to start off with a simplified definition for the average expected failure rate

$$\bar{\lambda}(t) = \frac{\Delta E}{\Delta t} \qquad (19)$$

where ΔE is the expected fractional units that fail in the time interval Δt. Then in the limit as Δ becomes infinitesimally small, we write the failure rate as

$$\lambda(t) = \lim_{\Delta\to 0}\frac{\Delta E}{\Delta t} = \frac{dE}{dt} \qquad (20)$$

Let's now look at an oversimplified parametric aging power law form for the three stages of creep

$$\Delta\varepsilon = \varepsilon_o t^N \qquad \begin{cases} N < 1 & Stage\ 1 \\ N \geq 1 & Stage\ 2 \\ N > 1 & Stage\ 3 \end{cases} \qquad (21)$$

There are numerous time dependent creep models that are commonly used that have more complex forms and are better suited to model creep. For example, Eq. 21 has different stresses that affect the creep slope in Figure 5 and 6. This particular model is oversimplified, but it is roughly capable of modeling all three stages of creep shown in Figure 5. This oversimplified power law form is very instructional as there are numerous similar power physics of failure aging laws (such as metal fatigue S-N curves, capacitor voltage breakdown and so forth) of this type in physics of failure applications. The three stages of creep are shown in the Figure 5 and 6.

When N is between 0 and 1 it models Primary Stage 1, when N is 1 it models Secondary Stage 2, and when N>1 it models Tertiary Stage 3. The creep rate is

$$\frac{d\Delta\varepsilon}{dt} = \varepsilon_o N\ t^{N-1} \qquad (22)$$

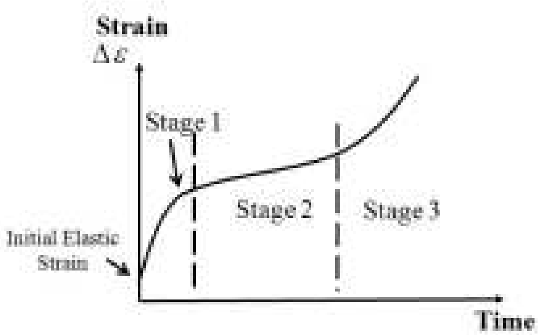

Figure 5 Creep curve with all three stages

Now using the different power law values for creep, we can plot the creep rate curve as shown in Figure 6 [8]. Interestingly enough the bathtub curve in Figure 1 has a similar shape to the creep rate curve shown. Note that Stage 2 of creep is not typically flat like the idealized bathtub curve (which likely may not be flat in the non idealized case).

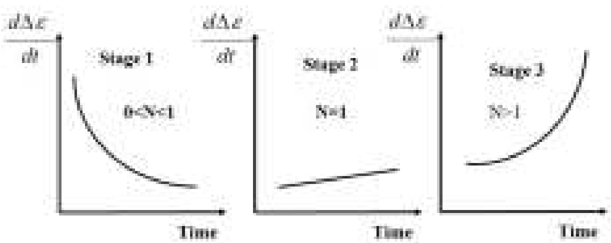

Figure 6 Creep rate power law model for each creep stage similar to the bathtubcurve in Figure 1

What we are thus tempted to do is try and merge the physics of creep to the statistical failure rate equations and make inferences. We will see that it is insightful.

Let us start by saying that for any creep phase, we can have a parametric failure corresponding to the parametric failure threshold t_f, so that the time to failure in general will be given by

$$\Delta\varepsilon_f(t) = \varepsilon_o t_f^{\,N} \tag{23}$$

Here we assume the following parametric treatment. In testing for example, when a device passes the failure threshold, $\Delta\varepsilon_f$, it will correspond to a time t_f, We count the device as a failure and proceed to perform some sort of traditional reliability catastrophic type of analysis to find its failure distribution and failure rate based on the times to failure for each device that passes the creep threshold. Now we have no idea of the life test parametric distribution.

However, in this discussion we would like to proceed and make inferences from the aging law on how it influences the statistics. The expected fraction of devices that will fail ΔE ($\Delta\varepsilon_f$(t)) in the time interval Δt then must be a function of the aging law so that the failure rate as we have defined it above is

$$\lambda(t) = \frac{dE}{d(\Delta\varepsilon_f(t))}\frac{d(\Delta\varepsilon_f(t))}{dt} = g(E)\,\varepsilon_o N\,t_f^{N-1} \tag{24}$$

Here we have let

$$g(E) = \frac{dE}{d(\Delta\varepsilon_f(t))} \tag{25}$$

If we assume a Weibull distribution for the parametric failure rate, we can now can make some observations. By direct comparison to the traditional Weibull parameters between Equations 17 and 24 we conclude

$$N = \beta \quad \text{and} \quad g(E)\varepsilon_o = (1/\alpha)^{\beta} \tag{26}$$

So in this model if N is between 0 and 1, say β=N=1/2, indicating that creep is in the Creep Primary Stage 1, then we are also in the infant mortality region. This is reasonable as it indicates early failure. If N=1, then we have a constant creep rate which is in the Secondary Creep Stage 2. This is also associated with the steady state region of the bathtub curve as *β=1*. Finally, if N=*β>1,* we reach the Tertiary Creep Stage 3 we are in wearout phase of the bathtub curve. Therefore, the physics for creep rate matches the statistics reasonably well,

> ***Essentially we have made direct comparisons between the creep rate in Figure 6 (Eq. 22) and the failure rate in Eq. 17, finding that N~β. Therefore, it is likely that for numerous aging power laws, when carefully modeled as we have done for the creep rate, can be directly tied to the value of the Weibull Beta. We have now connected the Weibull Model to physics of failure aging power laws.***

In catastrophic analysis, it is customary to assign alpha to a value of the aging equation, for example

$$\alpha = t_f = \left(\frac{\Delta\varepsilon_f}{\varepsilon_o}\right)^{1/N} \tag{27}$$

This is a number so we can just keep it in mind. It is evaluated at the failure time for parametric failure.

3.2 What Have We Shown?

We have used the creep physics of failure power aging law to connect the Weibull beta to the creep aging law exponent. As well, we can estimate the Weibull eta from the physics of failure aging law based on a sample population enough to satisfy Eq. 25-27. Although we have done this for the creep example, this holds for other physics of failure power aging law having the creep form. Therefore, we can predict the parametric failure rate based on a small sample size which is large enough to estimate the physics of failure aging law and the requirements of Eq. 25-27. We do not have to actually achieve failure on life test for this prediction in the parametric case since the aging law can predict the life. Although primarily for parametric failure rates, the catastrophic case is logically linked in a similar manner when most of the lifetime is dominated by a power aging law. Here, the intuition for Beta value can assist in Weibayes analysis.

4 ARRHENIUS ACTIVATION AND LOG(TIME) AGING

The last aging law is Arrhenius activation. This popular model has been shown to cause parametric Log(time) aging rates [2, 5] which we have shown can be related to the lognormal distribution. Due to page limitation in this article, this model is not reviewed here. However we would like to direct the interested reader to references 2, 5, 6 and 9 as these provide an excellent overview of this aging law.

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Alec Feinberg is the founder of DfRSoft. He has a Ph.D. in Physics and is the principal author of the book, Design for Reliability. Written in an industrial environment, it is a very practical book and has a very extensive approach to the DfR process using a stage gate method since products are develop in phases. Alec uses this method in his reliability training classes found on the DfRSoft website. Alec is also the principal developer for DfRSoftware which is the most thorough reliability tool currently available and is also used to accelerate learning in his training classes. Alec industrial experience has allowed him to provide extensive reliability engineering services in diverse industries (AT&T Bell Labs, TASC, M/A-COM, Tyco Electronics, and Advanced Energy) for over 35 years on solar, thin film power electronics, defense, microelectronics, aerospace, wireless electronics, and automotive electrical systems. He has provided training classes in Design for Reliability & Quality, Shock and Vibration, HALT, Reliability Growth, and Electrostatic Discharge. Alec has presented numerous technical papers and won the 2003 RAMS Alan O. Plait best tutorial award for the topic, "Thermodynamic Reliability Engineering." Alec has a new book, Thermodynamic Degradation Science: Physics of Failure, Accelerated Testing, Fatigue, and Reliability Applications (John Wiley& Sons, 2016).