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# Chi-Squared Accelerated Reliability Growth (CARG) Model

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*SUMMARY & CONCLUSIONS*

A Chi-Squared Accelerated Reliability Growth (CARG) model has been developed as a new method for single- and multi-stress level reliability growth life data analysis. The model is relatively easy to apply and is very practical. The CARG method is appropriate when an exponential distribution can be assumed. The chi-squared distribution has been used as a traditional method of identifying reliability confidence bounds for the exponential failure lifetime behavior of components, assemblies, and systems and is often extended to accelerated life test data analysis. The distribution is key for assessment when observance of few or even zero failures occur in accelerated testing for estimates on reliability at a statistical significance level. It is therefore natural to consider using the chi-squared method in the application of accelerated reliability growth data analysis. Using the statistic, the model is demonstrated on a manufacturing data set consisting of single accelerated stress and multi-accelerated stress tests. Reliability growth predictions show good agreement with the product's field data.

Companies today can have repetitive testing on products. Often identical tests are performed on each new product such as high temperature, vibration test, temperature cycle, humidity, etc. Each test may only have one stress level, with multiple groups tested. Fixes can be incorporated without time to re-test, and the reliability engineer has few options for quantifying the reliability growth that has likely occurred.

The CARG model offers a way to estimate accelerated reliability growth and it incorporates a fix-effectiveness factor. In the single-stress application, such as a single high temperature test, multiple groups might be tested and for each test group, the acceleration factor and fix effectiveness are applied, the CARG output provides estimates of the initial and final reliability, then the total reliability and reliability growth is found for all groups under this one high temperature stress. For multi-stress testing, the product may be tested for alternate types of stresses such as vibration test and temperature cycle in addition to the high temperature. The reliability growth is then found for each group and test type in a similar manner. Then the total reliability growth can be assessed for the product amongst all test types. Also shown are how growth alphas can be obtained. This not only provides growth quantification, but can be compared to traditional growth methods. Finally, in addition to the CARG analysis model, a CARG planning model is provided.

## *1* INTRODUCTION

Reliability engineers and managers need to justify their expensive chi-squared accelerated testing programs. As well they are incorporating fixes and may or may not have time to retest. CARG allows for reliability growth assessment with or without retest. CARG is likely the best choice when analyzing reliability growth achieved in accelerated chi-squared testing. It simply makes sense when such growth tests are analyzed. While currently this method is only used by a few of our customers, it may be anticipated that it will become popular since it has practical application to numerous areas in accelerated testing.

Furthermore the traditional Duane [1] or Crow/AMSAA [2] analysis method have a number of shortcomings:

- Growth assessment to a zero failure test result is problematic for the models
- Statistical significance is not intrinsic in the basic models
- The models don't not lend themselves easily to multi-growth test analysis [3]

The approach here overcomes these issues quite easily. The main supposition is that the test data is assumed to be exponentially distributed. This assumption while sometimes crude is the logical choice and is therefore often used extensively in chi-squared accelerated testing since failures in time are few in number compared to the sample population (i.e. not enough failure data to establish and other distribution). Many complex components and systems spend most of their lifetimes in this portion of the Bathtub Curve.

In this instance, a method for estimating the reliability growth using the chi-squared statistic is presented. The method appears flexible, straight forward, and allows for incorporating an estimate for fix effectiveness. A CARG planning model is also provided to help in accelerated stress test design for planning of test times and sample sizes.

## 2 CARG MODEL

The CARG model is presented for three test scenarios, case of a single accelerated stress test, case for single stress accelerated test with multiple tests groups, case for multiple accelerated stress test types and multiple tests groups. An example is provided for each as well.

### *2.1 Case for a Singe Accelerated Stress Test*

In the simplest case of a single accelerated stress test, the reliability growth achieved is simply given by

$$\Delta\lambda^1_{1\,Growth} = \lambda^1_{1\,initial}\,(\chi^2(\gamma, Y^1_1), N^1_1\,A^1_1\,t^1_1) - \lambda^1_{1\,final}\,(\chi^2(\gamma, f^1_1 Y^1_1), N^1_1\,A^1_1\,t^1_1)$$

Here terms are defined as:

$V^{Stress\,Type}_{Test\,Number}$ = Parameter indexing, superscript is stress type, subscript is test number

$\lambda$ = single-sided upper confidence bound on the failure rate

$\gamma$ = Chi-square confidence value

$Y$ = Number of failures for the stress test

$f$ = fix effectiveness factor that ranges between 0 and 1 such that $f$=(100-P)/100 where P is the percent estimate of fix effectiveness (typically set at 75%).

N = Number units tested

A=Acceleration factor for the test

t=Test time

The fix effectiveness factor should take into account both type A and B failure modes. A modes are failure modes that when seen during the test, no corrective action will be taken. This accounts for all modes for which management determines that it is not economically or otherwise justified to take a corrective action. B modes are those for which corrective action is implemented. Note that in the instance of retest verification of B failure modes, and when A failure modes are absent, the fix effectiveness factor is not required so that $f^1_1 Y^1_1$ can be replaced by $Y^1_{1\,Final}$ .

### *2.2 Case for a Singe Accelerated Stress Type and Multiple Tests*

In the next case of multiple T tests groups of the same stress type S=1, the reliability growth achieved is

$$\Delta\lambda^1_{Stress T\ Growth} = \lambda^1_{Iinitial}\,\{\chi^2(\gamma, \sum_{T=1}^{n} Y^1_T), \sum_{T=1}^{n} N^1_T A^1_T\,t^1_T\}$$

$$-\,\lambda^1_{Final}\,\{\chi^2(\gamma, \sum_{T=1}^{N} f^1_T Y^1_T), \sum_{T=1}^{N} N^1_T A^1_T\,t^1_T\}$$

### *2.3 Case for Multiple Accelerated Stress Types and Multiple Tests*

In the next case of multiple T tests groups and a number of different types of accelerated S stress tests, the total reliability growth achieved is simply

$$\Delta\lambda^{All}_{Stress\ Growth} = \sum_{S=1}^{K} \lambda^S_{Initial} - \sum_{S=1}^{K} \lambda^S_{Final}$$

## 3 DUANE TYPE GROWTH PARAMETER ESTIMATION

The result in each case will typically provide two data points on a growth plot for the initial and final failure rates for each accelerated stress test, and the total growth achieved for all test. If we chose, we can estimate the Duane or Crow/AMSAA type of growth parameter using the accelerated stress test using the relationship

Here β-1 is given by the slope of the log-log plot over the test time. The growth alpha=β-1 is often the value quoted in reliability growth assessment. In this way, a reliability growth alpha can be applied via the traditional growth equation so that the user has an estimate of the capability for each stress test and if interested one can in fact compare results to historical growth alphas.

## 4 CARG APPLICATION EXAMPLE

Here CARG has been applied to the data set for accelerated testing that was performed on a small electronic assembly displayed in Table 1. Analysis of the data was done at the 60% and 90% confidence level This product was subjected to accelerated stress tests of humidity, temperature shock, and vibration. The table displays the sample size, test time, acceleration factor, number of failures, and assumed fix effectiveness factor. Note the fix effectiveness factor was set to 75%. Often either 75% of 70% are use as conservative estimates.

| **Humidity Test** | **Sample Size** | **Test Time** | **Accel. Factor** | **Num. Fail Test** | **Fix Effec. Factor (%)** |
|---|---|---|---|---|---|
| Humidity Group 1 | 30 | 100 | 55 | 0 | 75 |
| Humidity Group 2 | 20 | 100 | 55 | 2 | 75 |
| Humidity Group 3 | 25 | 100 | 55 | 0 | 75 |
| Humidity Group 4 | 40 | 100 | 55 | 0 | 75 |
| Humidity Test Summary | 115 | 400 | 55 | 2 | 75 |

*Table 1A. Humidity Accelerated Test*

| **Temperature Shock** | **Sample Size** | **Test Time** | **Accel. Factor** | **Num. Fail** | **Fix Effec. %** |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| Temp Shock Group 1 | 22 | 240 | 13 | 0 | 75 |
| Temp Shock Group 2 | 35 | 240 | 13 | 1 | 75 |
| Temp Shock Group 3 | 22 | 240 | 13 | 1 | 75 |
| Temp Shock Group 4 | 30 | 240 | 13 | 1 | 75 |
| Temperature Shock Test Summary | 109 | 960 | 13 | 3 | 75 |

*Table 1B. Temperature Shock Accelerated Test*

| Vibration | Sample Size | Test Time | Accel. Factor | Num. Fail | Fix Effec. % |
|---|---|---|---|---|---|
| Test Group 1 | 70 | 18 | 1460 | 3 | 75 |
| Test Group 2 | 65 | 18 | 1460 | 3 | 75 |
| Test Group 3 | 60 | 18 | 1460 | 4 | 75 |
| Test Group 4 | 100 | 18 | 1460 | 5 | 75 |
| Vibration Test Summary | 295 | 72 | 1460 | 15 | 75 |

*Table 1C. Vibration Accelerated Test*

| | Total Samples | Total Test Time | Total Fail | Avg. Fix Effec-tiveness |
|---|---|---|---|---|
| All Test Summary | 519 | 1432 Hrs | 20 | 75% |

*Table 1D. Summary of Tables 1A, 1B, and 1C*

As an example, for the Temperature Shock Group (Table 1B), the case for this single accelerated stress test is displayed in Figure 1. Note that no failures were observed in Group 1, as indicated by the flat line. So for that, there was no opportunity for growth.

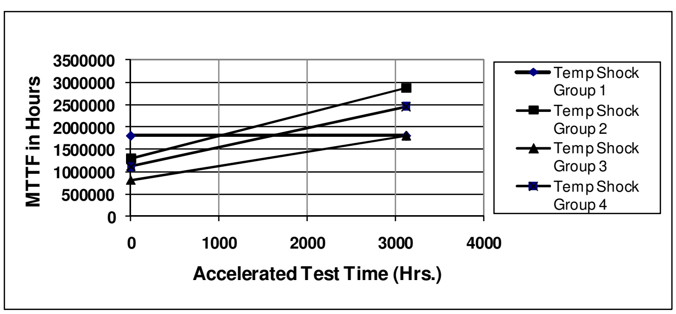


*Figure 1: CARG for Temperature Shock Test Groups*

In the full case of multiple tests groups and a number of different types of accelerated S=1, 2, and 3 stress tests of Humidity, Temperature Shock, and Vibration testing, the CARG graphical results are displayed in Figure 2.

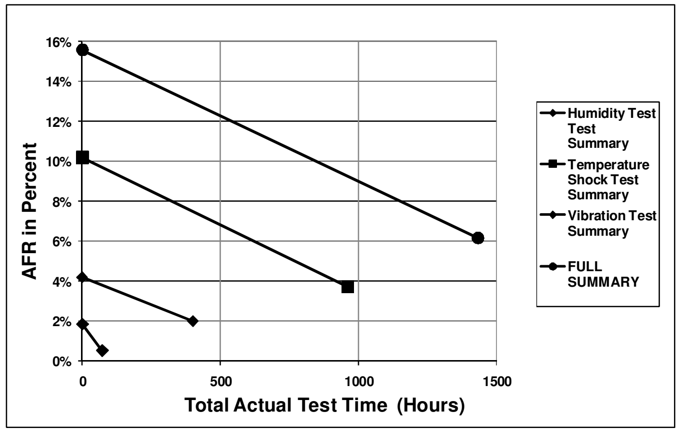


*Figure 2: CARG results for multiple testing with multiple groups per test*

This analysis in Figure 1 was performed at the 60% confidence level and the reliability growth results are obtained in terms of the device's AFR (Annual Failure Rate) where

$$AFR=1-Exp(-8760/MTTF) \quad (5)$$

Table 2 displays a summary at the 60% and 90% chi-squared confidence levels. Field data indicated that the product's AFR was between these confidence limits yielding CARG AFR estimates of 6.2% and 12.4%. Per Equation 4, the growth alphas were obtained and are given in the table. Note that the most growth achieved was in vibration testing as indicated by the growth alphas having the largest value ranging from 0.25 to 0.296. There the percent of growth achieved was estimated to be between 65.4% and 73.7%.

**Table 2** CARG analysis results of the data in Table 1

| Test | CL | Initial AFR | Final AFR | Growth % | Growth Alpha |
|---|---|---|---|---|---|
| All Tests Combined | 60% | 15.6% | 6.0% | 60.3% | 0.134 |
| | 90% | 23.7% | 12.4% | 47.7% | 0.098 |
| Humidity Test | 60% | 4.2% | 2.0% | 52.4% | 0.124 |
| | 90% | 7.1% | 4.2% | 40.8% | 0.089 |
| Temperature Shock | 60% | 10.2% | 3.7% | 63.7% | 0.152 |
| | 90% | 15.8% | 7.7% | 51.3% | 0.11 |
| Vibration | 60% | 1.9% | 0.5% | 73.7% | 0.296 |
| | 90% | 2.4% | 0.83% | 65.4% | 0.25 |

Historical growth alphas are not readily available for this type of electronic assembly. However, for complex military systems, Crow [4] has quoted values between 0.24 and 0.6 for such systems as Navigation systems, Helicopters, Missiles etc. The average value found was 0.4. In this case, a more modest growth alpha between 0.1 and 0.3 is obtained, which is perhaps related to a simpler assembly with less growth opportunities.

Finally, with the AFR values, an estimate of the cost savings from the CARG model can be obtained. As mentioned earlier, reliability managers have the difficult task of quantifying the benefits of their accelerated testing programs. Thus, Table 3 is a rough scenario on how one might quantify such benefits for a program using the CARG method. Saving estimates are displayed in the last column for this product. Costs saving estimates are between $3300 and $4000 per 1000 units shipped.

| Conf. | Unit Cost | Initial AFR | No Growth Warr-antee Loss Per 1000 | Final AFR | Growth Warr-antee Loss Per 1000 | Savings Per 1000 |
|---|---|---|---|---|---|---|
| 60% | $35 | 15.6% | $5,469 | 6.2% | $2,170 | $3,299 |
| 90% | $35 | 23.7% | $8,295 | 12.4% | $4,340 | $3,955 |

Table 3 CARG warrantee cost saving per 1000 units

## 5 CARG PLANNING MODEL

Historically, reliability growth planning is often included in modeling. To this end, CARG reliability growth planning can be done using the Duane equation

$$\lambda_F = \lambda_I \, t^{\beta-1} \tag{6}$$

The main difference of course is to introduce a modification of a chi-squared interpretation for the initial failure rate, i.e.

$$\lambda_I = \lambda_{initial} \left(\chi^2(\gamma, Y \;), N \;\; A \;\; t \;\;\right) \tag{7}$$

As an example, let's use the known values in Table 1 and 2 with an initial AFR of 15.6% and the final AFR growth objective that was observed of 6.2%. Thus, using Equation 5, and the current overall growth alpha of 0.134 (60% conf. level), the first planning objective can be found, the total test time t, which comes out to about 1400 hours (as expected, see Table 1D). At this point in planning, one should have some idea of the allocation times amongst stress tests and acceleration factors. With that in mind, the values for $N^S$ are needed for planning purposes for each S stress test and some estimate of $Y^S$. The wise choice is to plan testing at both 60% and 90% with some conservative Y-value estimates and solve for N for each stress test. Such planning may be helpful in CARG growth test design.

## 7. *BIOGRAPHY*

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Alec Feinberg has a Ph.D. in Physics and is the principal author and editor of the book, *Design for Reliability [3]*. Alec's started the company, DfRSoft. At DfRSoft he has developed design for reliability software that is used worldwide and he works as a consultant. Alec has provided reliability engineering services in all areas of reliability and on numerous products in diverse industries that include solar, thin film power electronics, defense, microelectronics, aerospace, wireless electronics, and automotive electrical systems. He has provided training classes in Design for Reliability, Shock and Vibration, HALT, Reliability Growth, Electrostatic Discharge, Dielectric Breakdown, DFMEA and Thermodynamic Reliability Engineering. He previously worked for Advanced Energy, Tyco Electronics/MACOM, TASC, and AT&T Bell Laboratories. Alec has presented numerous technical papers and won the 2003 RAMS Alan O. Plait best tutorial award for the topic, Thermodynamic Reliability Engineering.

## 6. *FINAL CONCLUSIONS*

We have described a Chi-Squared Accelerated Reliability Growth model as a new method for single- and multi-stress level reliability growth life data analysis method. The model is relatively easy to apply, is very practical, and was demonstrated on a manufacturing data set. Results showed good agreement with field data's AFR which was found to be within the 60% and 90% confidence bond estimates for the product. A reliability growth alpha was also provided by using the Duane growth equation so that the user has an estimate of the capability for each stress test and if interested, can compare to historical growth alphas. Finally, a CARG planning model was proposed using the Duane equation with a chi-squared interpretation of the initial failure rate.

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