RELIABILITY PREDICTIVE MODELING

Defining The Future

11.1 INTRODUCTION

Reliability predictions play a critical role in the Design for Reliability (DfR) process during product development. Reliability predictions provide early estimates of the design complexity that relates to the product reliability. When a prediction method is accompanied by appropriate realism factors, it can also provide excellent estimates of the expected reliability in actual use conditions.

Reliability predictions are generally made for steady-state operation. The steady-state portion of life is discussed in Chapter 8 and it is depicted in this chapter by the bathtub curve in Figure 8.6. The steady-state region is modeled with a constant failure rate. Reliability predictions can be performed for any aspect of the bathtub curve or for any other realistic characteristics, but this chapter is focused on reliability predictions for this constant failure rate region.

Reliability predictions can be used for many purposes during product development. Typical applications include:

- determining the feasibility of meeting a reliability requirement or a goal;
- monitoring the complexity during the development process;
- estimating the expected rate of failures for the associated life cycle cost;
- estimating the failure rates that support design trade-off evaluations;
- estimating failure rates for calculating failure rate dependent characteristics such as maintainability or testability;
- estimating the failure rates for a Failure Modes and Effect Analysis (FMEA);
- supporting a customer-requested evaluation; and
- providing the failure rate expectations for various conditions (e.g., thermal extremes or user environments such as mobile and fixed ground sites).

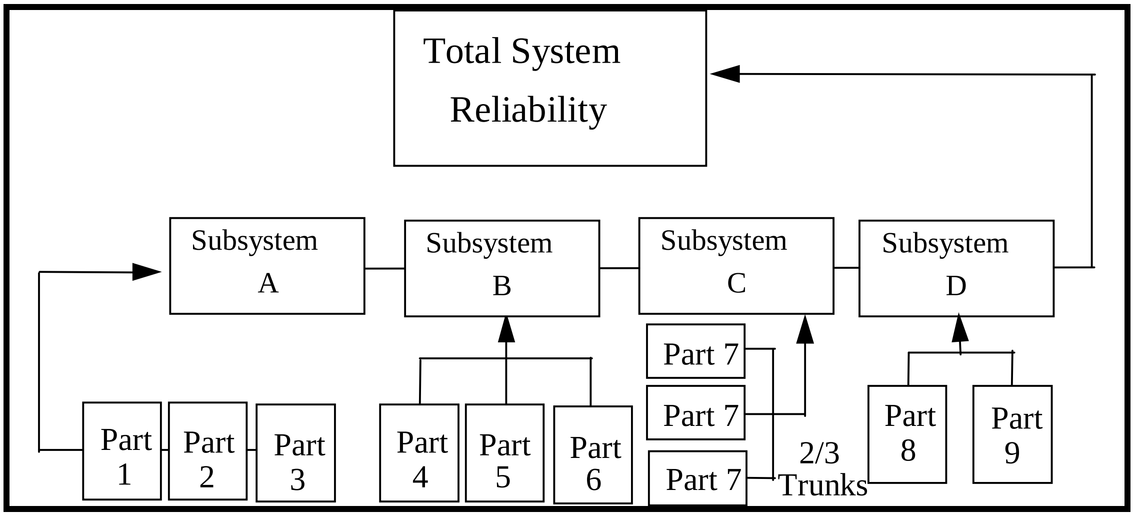

Figure 11.1 System Reliability Block Diagram Representation.

11.2 SYSTEM RELIABILITY MODELING

Most common reliability predictions use a "bottom-up" approach by estimating the failure rate for each element and then combining the failure rates for the entire assembly (see Figure 11.1). In the block diagram configuration, the system is broken down to the lowest elements of interest. Figure 11.1 illustrates a number of important block diagram representations. Here, the system failure rate is the sum of the individual subsystems A, B, C, and D. The subsystems are in a series configuration; if any subsystem fails it results in a system failure. There are three traditional types of block diagrams represented. The subsystems shown in Figure 11.1 have elements that are purposely configured to illustrate each type. Subsystem A consists of parts 1, 2, and 3 that are in the series configuration (if any part fails then subsystem A fails). Subsystem B consists of parts 4, 5, and 6 in a parallel configuration. Similarly, subsystem D has parts 8 and 9 in parallel. Parallel subsystems indicate redundancy. For example, in Subsystem B, any two parts can fail without failure to the subsystem. All three would have to fail for Subsystem B to fail. Lastly, Subsystem C has elements that appear to be in parallel. However, the 2/3 trunks is used to indicate that as long as two of the three elements are working, the subsystem is operational.

Once the failure rate for each subsystem is determined, the results can be rolled-up into a reliability prediction for the system itself. This is the bottom-up approach. In each type of configuration, the method for determining the failure rate is different.

11.2.1 Series Systems (Subsystem A)

Reliability predictions are generally made for steady-state operation. Therefore, usually the exponential distribution is assumed (see Figure 8.7). The reliability function for the exponential distribution

$$R(t) = e^{-\lambda t}$$

is the probability of a component surviving to time t. Therefore, for a system made up of n independent components, where any component failure causes a system failure, the probability of survival for the whole system is

$$R_{system} = R_1 R_2 R_3 R_4 \cdots R_n$$

or

$$e^{-\lambda_{SYSTEM} t} = e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t} \cdots e^{-\lambda_n t} = e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n)\ t}$$

Therefore,

$$\lambda_{System} = \lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n$$

The failure rate of the system is simply the sum of the failure rates of the individual devices.

EXAMPLE 11.1 Failure Rate of Subsystem A.

The failure rates of parts 1, 2, and 3 are 0.1, 0.3, and 0.5 hours^{-1}, respectively. Determine the failure rate for Subsystem A.

SOLUTION: Since the total failure rate is simply the sum of the individual rates, the failure rate of Subsystem A is 0.9 hours^{-1}.

11.2.2 Parallel Systems (Subsystem D)

Parallel systems indicate redundancy. The simplest example of redundancy is a situation in which two elements are in a parallel reliability configuration. Here, it is simplest to work with failure probabilities R'=(1-R) (indicated with a prime). Then the probability of a system failing consisting of two parallel elements is

$$R'_{System} = R'_1\ R'_2$$

or

$$1-R_{System} = (1-R_1)(1-R_2)$$

Solving this gives

$$R_{System}=R_1+R_2-R_1R_2$$

Substituting in the reliability function for the exponential distribution yields

$$R_{System}=e^{-\lambda_1 t}+e^{-\lambda_2 t}-e^{-\lambda_1 t}e^{-\lambda_2 t}=e^{-\lambda_1 t}+e^{-\lambda_2 t}-e^{-(\lambda_1+\lambda_2)t}$$

The right-hand side is a complicated function and R_{system} cannot be put into simple exponential form. Therefore, the actual system failure rate cannot easily be defined by the exponential distribution. However, both sides can be integrated over all time as

$$\int_0^\infty R_{System}(t)dt=\int_0^\infty [e^{-\lambda_1 t}+e^{-\lambda_2 t}-e^{-(\lambda_1+\lambda_2)t}]dt=\int_0^\infty [e^{-\lambda_1 t}dt+\int_0^\infty e^{-\lambda_2 t}dt-\int_0^\infty e^{-(\lambda_1+\lambda_2)t}dt$$

The left-hand side is the system's MTTF (see Section 8.2)

$$MTTF_{System}=\int_0^\infty R(t)\ \ dt=\frac{1}{\lambda_{eff}}$$

and its inverse, λ_{eff}, is denoted as an effective failure rate for the system. The integrals on the right-hand side of the equation have constant failure rates. For example

$$\int_0^\infty e^{-\lambda_1 t}\ \ dt=\frac{1}{\lambda_1}$$

Then a system's MTTF for two elements in parallel is

$$MTTF_{System}=\frac{1}{\lambda_{eff}}=\frac{1}{\lambda_1}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1+\lambda_2}$$

EXAMPLE 11.2 Failure Rate of Subsystem D

The failure rates of parts 8 and 9 in Subsystem D are 0.25 and 0.2 hour^{-1}, respectively. Determine the effective failure rate of Subsystem D.

SOLUTION: The failure rate for Subsystem D is found from

$$\frac{1}{\lambda_{eff\ D}}=\frac{1}{\lambda_8}+\frac{1}{\lambda_9}-\frac{1}{\lambda_8+\lambda_9}$$

Substituting in the failure rate values gives

$$MTTF_D=\frac{1}{\lambda_{eff\ D}}=4+5-2.22=6.78$$

Therefore, the effective failure rate of Subsystem D is 0.147 hour^{-1}.

11.2.3 Reducing Redundancy in Modeling

In general, the above approach can be used for a subsystem with three items in parallel, such as Subsystem B in Figure 11.1. However, this can become cumbersome. Alternately, redundancy can be reduced and allowing the use of the formula above. For example, modeling redundancy of Subsystem B can be reduced as shown in Figure 11.2.

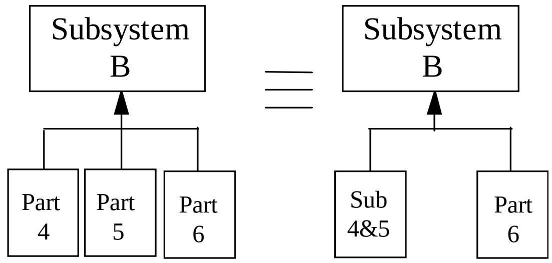

Figure 11.2 Reducing redundancy in modeling.

Here, combining parts 4 and 5 creates a new subsystem. The effective failure rate of this subsystem is that of two parts in parallel given by

$$\frac{1}{\lambda_{eff\ Sub\ 4\&5}} = \frac{1}{\lambda_4} + \frac{1}{\lambda_5} - \frac{1}{\lambda_4 + \lambda_5}$$

This reduces the problem of determining the failure rate for the overall subsystem. The failure rate for Subsystem B is now reduced to a problem of determining the failure rate for two elements in parallel and is given by

$$MTTF_D = \frac{1}{\lambda_{eff\ B}} = \frac{1}{\lambda_{eff\ Sub\ 4\&5}} + \frac{1}{\lambda_6} - \frac{1}{\lambda_{eff\ Sub\ 4\&5} + \lambda_6}$$

EXAMPLE 11.3 Failure rate of Subsystem B
The failure rates of parts 4, 5, and 6 in Subsystem B are 0.2, 0.4, and 0.25 hour^{-1} respectively. Determine the effective failure rate for subsystem B.

SOLUTION: Combining parts 4 and 5 above, into a subsystem as above yields

$$\frac{1}{\lambda_{eff\ Sub\ 4\&5}} = 5 + 2.5 - 1.66 = 5.83$$

This gives an effective failure rate of 0.171 hour^{-1}. Then the effective failure rate of Subsystem B is found from the equation above as

$$\frac{1}{\lambda_{eff\ \ Subsystem\ B}} = 5.83 + 4 - 2.375 = 7.46$$

This gives the effective failure rate of subsystem B of 0.134 hour^{-1}.

11.2.4 Modeling k of n Subsystem Elements

The probability of at least k out of n identical elements working in a subsystem is given in probability theory. In terms of the probability of success R this is

$$R_{System} = \sum_{i=0}^{n-k} \frac{n!}{i!(n-i)!} (R_A)^{n-i} (1 - R_A)^i$$

EXAMPLE 11.4 FAILURE Rate of Subsystem C and the System
Determine the effective failure rate for Subsystem C in which at least 2 out of 3 items must be working. The failure rate of item 7 is constant and is 0.2 hours^{-1}. Then using this result and that of Examples 11.1 through 11.3, determine the effective failure rate of the system.
SOLUTION: Using the above formula yields

$$R_{Subsystem\ C} = \frac{3!}{0!(3-0)!}(R_7)^3(1) + \frac{3!}{1!(3-1)!}(R_7)^2(1-R_7) = (R_7)^3 + 3(R_7)^2(1-R_7)$$

In terms of the exponential reliability function this is

$$R_{Subsystem\ C} = e^{-3\lambda_7 t} + 3e^{-2\lambda_7 t}(1 - e^{-\lambda_7 t})$$

Simplifying this is

$$R_{Subsystem\ C} = 3e^{-2\lambda_7 t} - 2e^{-3\lambda_7 t}$$

Integrating both sides of this equation over all time gives

$$MTTF_{SubsystemC} = \frac{1}{\lambda_{eff\ \ C}} = \frac{3}{2\lambda_7} - \frac{2}{3\lambda_7} = \frac{5}{6\lambda_7}$$

Substituting in the failure rate for part 7 of 0.2 hours^{-1} gives

$$\lambda_{eff} = \frac{6\lambda_7}{5} = \frac{6(0.2)}{5} = 0.24$$

From Examples 11.1 through 11.4, the effective failure rates for subsystems A, B, C, and D have now been determined as 0.9, 0.134, 0.24, and 0.147 hours^{-1}, respectively. Therefore, the total effective failure rate of the system can be obtained from the sum as 1.42 hours^{-1} (or a 0.704-hour system MTTF).

11.2.5 Other Configurations and Repair/Availability

This section has covered a number of typical block diagram configurations. Other configurations exist. Appendix A covers a number of k out of n type redundant configurations. Additionally, the effective failure rate for a redundant system can be extended with repair. That is, when one item fails in a redundant system, there is an opportunity to repair it before subsystem failure. Therefore, the effective failure rate needs to incorporate the possibility that the unit will be fixed and back on-line with full redundancy again available before subsystem failure. Appendix B tabulates these types of situations. Additionally, sometimes redundancy is achieved by having units on standby waiting to be substituted for a potential component failure. In this case, the effective failure rate is a function of the switching mechanism and the number of active units. This is also described in Appendix B.

For very complex systems that require frequent repair, often the reliability metric of interest by a customer, is expected equipment availability. This is described in Appendix C.

11.3 CUSTOMER EXPECTATIONS

Customer expectations for reliability predictions can vary quite significantly, especially with a global market that encompasses a wide range of applications. Therefore, it is important to be prepared to respond to a remarkably wide range of expectations. It is common for multiple methods to be performed during a development project. In fact, it would be unusual if only one reliability prediction was performed per project.

11.4 VARIOUS METHODS

There are many available methods for performing reliability predictions in our industry. Each method has some advantages and some weaknesses. This section will describe some of the frequently applied methods in sufficient detail, to understand what is involved in performing each method and what type of output is available. Many methods have been omitted out of necessity, to focus on predominant methods.

There are many excellent tools available. Expertise with a specific tool will substantially impact the user satisfaction and productivity, regardless of the tool or the prediction method. This chapter focuses on two specific methods:

- Military Handbook-217, (latest versions FN2), for both Parts Count and Detail Stress Methods to estimate Electrical and Electronic Parts, using an exponential failure rate, and
- Bellcore, (latest issue 6) for Methods I, II, and III assuming a serial model, using exponential failure rates of electrical parts.
-

An example that compares Military Handbook-217 to Bellcore is provided in this chapter. Some discussions are provided on related materials and techniques for:

- estimating system reliability where the system elements are configured in six typical redundancy relationships, using the relationships defined in Reference 1;
- estimating extremely complex redundancy using discrete event simulation techniques;
- estimating the acceleration factors using the Arrhenius relationship;
- estimating acceleration factors using a wide range of other application factors; and
- estimating the resulting probability based on a combination of a wide range of conditional probabilities.

11.4.1 Military Handbook-217 Predictive Methods

In this section, Military Handbook-217 is discussed, versions E, F-1, F-2, Parts Count and Detail Stress Methods to estimate failure rates of electrical and electronic parts, using an exponential distribution. Military Handbook-217 has the most internationally recognized methods. For example, the Russian standard for reliability predictions can be read, even for the reader who cannot read Russian.

Versions of Military Handbook-217 have been widely used for more than 30 years. The major advantage of the Military Handbook is the widespread application and experience many people have with some type of realism factors, from a comparison of past predictions to the actual experience under some specific use conditions. The latest revision is Notice 2 for Military Handbook-217F, *MILITARY HANDBOOK Reliability Prediction of Electronic Equipment.* The Military Handbook describes both a Parts Count and Detail Stress method. The Military Handbook also provides failure rates for many environmental conditions, covering the range from ground benign for continuous operation in comfort controlled condition to cannon launch for electronics.

The Parts Count Method makes assumptions for a representative thermal ambient, part complexity, and various electrical stresses. These assumptions simplify the effort to perform these evaluations. This simplification allows for early evaluations to be performed. The Parts Count Method is ideal if the design is at a very early stage or if the analysis labor is to be minimized. Section 11.4.3 provides an example of the Military Handbook-217 Parts Count Method for a commercial electric clock. A Detail Stress Method uses the specific parts complexity and the specific application stresses. This added detail requires more time for the collection of parts library information and application stresses. The advantage of the Detail Stress Method is that the output conclusions will reflect the specific conditions and include the effects of the thermal conditions on the failure rate. This is very helpful if the usage includes multiple conditions or if accelerated testing is planned. The primary limitation for either the parts count or the detail stress methods, is the availability of realism factors or experience comparing past reliability predictions with actual experience. Great care must also be exercised in selecting and interpreting the outputs in terms of the versions. For example, Version F, Notice 2, detail stress method has a much higher failure rate than Version F, Notice 1. Finally, note that the units used with Military Handbook failure rate is in Failures Per Million Hours (FPMH).

11.4.2 Bellcore Predictive Methods

Bellcore is the research group for AT&T. They created the Bellcore methods because they were not satisfied with the applicability of the Military Handbook methods for their commercial products or for their markets. They created the reliability prediction guidance documents for use on their products. Bellcore is intended for commercial (i.e., non-military) parts. The latest revision is Technical Reference TR-332 Issue 6, December 1997, called *Reliability Prediction Procedure for Electronic Equipment.* In this section, Bellcore Methods I, II, and III are discussed, assuming a serial model for exponential failure rates of electrical parts. Table 11.1 provides an overview of the differences between Bellcore prediction methods and Military Handbook-217.

Their document says one of the purposes is for the recommended failure rates to contain the appropriate realism experience. Most practitioners who have done extensive comparisons will advise you to confirm the realism on your own products in their application. In fact, Methods II and III address this issue.

Method I uses a very similar approach to the Military Handbook-217 Parts Count Method. It uses representative complexities, stresses, and one or more environments as the basis. It only considers three environments: controlled fixed ground, uncontrolled fixed ground, and mobile ground. It addresses four quality levels: 0, II, III, or I. Additional factors translate from the representative conditions to other specific conditions, if desired. A Bellcore Method I prediction for a commercial electric clock is provided in the next section. Method II is based on combining Method I predictions with data from a laboratory test performed in accordance with specific Bellcore test criteria.

Method III is for statistical predictions of in-service reliability based on field tracking data, collected in accordance with specific Bellcore criteria. The Bellcore failure rate output is in units of FITs, which is equivalent to failures per billion hours (see Chapter 8).

The primary limitation for Method I is the availability of realism factors or experience comparing past reliability predictions with actual experience. Great care must be exercised in selecting and interpreting the outputs in terms of the versions.

Table 11.1 Comparison of MIL-HDBK-217 and Bellcore Procedures

PREDICTION TYPE	CONDITIONS		
Typical Procedure	**Information**	**Environmental Factors**	**Quality Factors**
Military Handbook 217 (Parts count)	Assume +40°C op. temp. and 50% electrical stress	Ground benign, fixed, or mobile. Airborne, Inhabited cargo, Missile launch, etc.	Jantxv, Jantx, Jan Commercial, Plastic
Military Handbook217,(Detailed Stress)	Specify stress level each component	Ground benign, fixed, or mobile. Airborne, Inhabited cargo, Missile launch, etc.	Jantxv, Jantx, Jan Commercial, Plastic
Bellcore Method I, case 1 (Parts count)	No device burn-in or unit burn-in <1 hr, +40°C, 50% stress	Ground fixed or mobile, Airborne, Space based commercial.	Level 0, Level I Level II, Level III
Bellcore Method I, case 2 (Parts count)	No device burn-in and unit burn-in > 1 hr, +40°C, 50% stress	Ground fixed or mobile, Airborne, Space based commercial.	Level 0, Level I Level II, Level III
Bellcore Method I, case 3 (Parts count)	General case, anything other than +40°C, 50% stress	Ground fixed or mobile, Airborne, Space based commercial.	Level 0, Level I Level II, Level III
Bellcore Method II (Combined lab data & parts count)	Predictions are based on combined parts count and lab data	Ground fixed or mobile, Airborne, Space based commercial.	Level 0, Level I Level II, Level III
Bellcore Method III (Predictions from field tracking)	Prediction based on field tracking data	Ground fixed or mobile, Airborne, Space based commercial.	Level 0, Level I Level II, Level III

11.4.3 Example Military Handbook-217 versus Parts Count to Bellcore Method I

It may be helpful to provide an example that compares the MIL-HDBK-217 Parts Count and Bellcore Method I. This example is for a commercial electric clock. The failure rate prediction analysis is provided in Table 11.2, with the system reliability results given in Table 11.3.

Table 11.2 Comparison of MILITARY HANDBOOK-217 and Bellcore for a Clock

	Mil-HDBK-217F Notice 1 (GB, Parts Count)	Bellcore (GC, Method I)

Bill Of Space Material (BOM)	Qty	**Generic FR** (FPMH)	**pi Q**	**FR** (FPMH)	**Generic FR** (FITs)	**pi Q**	**FR** (FITs)
Electrical Elements:							
Electric motor, AC	1	1.60000	1	1.6000	500	2.5	1,250.0
Buzzer (piezo-electric crystal)	1	0.03200	2.1	0.0672	50	2.5	125.0
Switch, buzzer, on-off	1	0.00100	20	0.0200	15	2.5	37.5
Connector, AC power	1	0.01100	2	0.0220	10	2.5	25.0
Solder joints	6	0.00014	2	0.0017	5	2.5	75.0
Crimp joints	2	0.00026	2	0.0010	5	2.5	25.0
Electrical cord	1	note 1			note 1		
Mechanical Elements:		note 2			note 2		
Gears	6	note 2			note 2		
Knobs	3	note 2			note 2		
Sweep hands	3	note 2			note 2		
Clock face	1	note 2			note 2		
Software Elements:	0	note 2			note 2		
Operator Error Elements:	1	note 2			note 2		
Total Failure Rate				1.7119			1,537.5

Note 1: Disregarded in Note 2. Not covered by this method.

Table 11.3 Comparison of Failure Rates and Reliability Results for a Clock

Dimension	Military Handbook 217F	Bellcore
Failure Rate	1.7119 (in FPMH)	1,537.5 (in FITs)
AFR in failures/year	0.01500	0.01347
MTBF in hours	584,139	650,407
MTBF in years	66.68	74.25

11.4.4 Additional Methods and Techniques

Discussions on other methods and techniques are described below:

Rome Air Force Development Center (RADC) Toolkit

Many excellent references are available as guidance for calculating redundancy. The Rome Air Force Development Center Toolkit (see Reference 1) is good reference because it uses language that is widely applied by the reliability practitioners. The Toolkit is also relatively inexpensive. It is a good reference for terminology and DfR methods. The Toolkit explains six redundancy conditions and it provides the guidance for calculating the system failure rate based on any of the six situations. Three of the six disregard the effects of maintenance and the other three include the effects of maintenance.

Estimating Complex Redundancy Using Discrete Event Simulation Techniques

Sometimes an extremely complex redundancy situation occurs that cannot be evaluated using simple relationships. If the redundancy involves queuing conditions, simple relationships are not adequate. In very complex situations, discrete event simulation models are needed. A number of specific software tools are available to perform simulation predictions.

Estimating Temperature Acceleration Factors Using the Arrhenius Relationship

The Arrhenius relationship is commonly used to estimate temperature acceleration factors. Details are described in Chapter 9. This relationship provides a convenient comparison of the effects of temperature for any device/failure mechanism with known activation energy. The Arrhenius relationship requires that you have two temperatures of interest and knowledge of the activation energy for failure mechanism. The model provides the acceleration (or deceleration) based on the different temperature conditions. The limitation is the knowledge about the activation energy and correlation to the failure mechanism.

Estimating Acceleration Factors Using a Wide Range of Other Application Factors

There are other stress models besides Arrhenius. Other models that are a function of the stress and for a particular failure mechanism (metal fatigue, corrosion, electromigration, etc.) may provide more appropriate estimates on reliability (see Chapter 9). Often, if the mechanical engineering group has insight into stress acceleration factors as they may have worked out finite element model on thermal and mechanical stress situations. The limitation for finite-element modeling is the analysis time to create and evaluate specific models for each specific situation of interest.

Estimating Reliability Using Conditional Probabilities Methods

The statistics for combining probabilities is a well-known science and an overview has been provided earlier in this chapter. System reliability often requires detailed probability analysis when serial models cannot be used. In this case, fundamental probability mathematics is required. Such modeling will most likely require expertise in this area, as software solution will most likely be unavailable.

11.5 COMMON PROBLEMS

Any technical analysis can encounter problems while performing the analysis. Reliability predictive methods are no exception. This section describes a few of these problems.

A common question is "How do I use this prediction to improve the product"? While there are many answers to that question, the first question the analyst may ask is "Can I use this analysis for the intended purpose"? Obviously, this infers that a purpose guided your selection method before you started analysis. The analysis should be reviewed first to ensure that it would have the precision needed for the application. This includes not only the analysis assumption, but also any realism factors you plan to use with the analysis. Once that hurdle has been cleared, the ability to use it should be easy.

Another common question concerns the library for the parts and the application stresses. It is customary to spend more time collecting the library and stress application information than it takes to input all of the information into one of the prediction tools. People who have not performed extensive evaluations often overlook the library importance.

Still another common question is related to the realism for the various evaluations. Unfortunately, many people have serious misunderstandings about the realism of various evaluation methods. None of the available industry-standard methods contains realism guidance or expectations. As a user you should establish a realism approach for your own applications and intended usage. Some experiences have been published in the public domain for guidance, but there is no substitute for developing your own realism factors. For example, it is helpful to compare your original predictions to field results.

APPENDIX A

TABULATED k of n SYSTEM EFFECTIVE FAILURE RATES

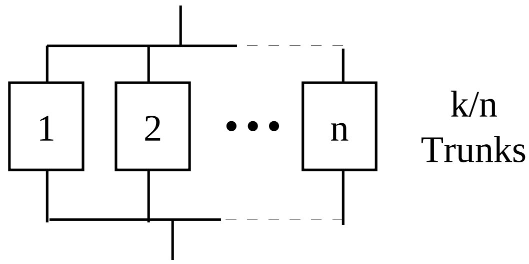

Figure A.1 k of n Block Diagram for Table A.1

For all units active with equal unit failure rates, and k out of n required for success as shown in Figure A.1, the effective failure rate is given by [1]

$$\lambda_{eff\ k/n} = \frac{\lambda}{\sum_{i=k}^{n} \frac{1}{i}}$$

Results from this equation are tabulated below. As an exercise verify the results in Example 11.4 using the above equation or table below.

Table A.1 Tabulated k of n system effective failure rate values

n	k	λeff		n	k	λeff
1	1	λ		5	4	(60/27) λ
2	1	(2/3) λ		5	5	5 λ
2	2	2 λ		6	1	(60/147) λ
3	1	(6/11) λ		6	2	(60/87) λ
3	2	(6/5) λ		6	3	(60/57) λ
3	3	3 λ		6	4	(60/37) λ
4	1	(12/25) λ		6	5	(60/11) λ
4	2	(12/13) λ		6	6	6 λ
4	3	(12/7) λ		7	1	(140/363) λ
4	4	4 λ		7	2	(140/223)λ
5	1	(60/137) λ		7	3	(140/153) λ
5	2	(60/77) λ		7	4	(420/319) λ
5	3	(60/47) λ		7	5	(210/107) λ

APPENDIX B

REDUNDANCY EQUATION WITH AND WITHOUT REPAIR

This appendix contains tabulated redundancy approximation given in Reference 1.

These equations have the following notations: $\lambda_{k/n}$ is the effective failure rate of the redundant configuration where n of k units are required for success, n is the active on-line units, λ is the failure rate of an individual on-line unit (failures/hour), μ is the repair rate (μ=1/Mct where Mct is the mean corrective maintenance time in hours), and P is the probability switching mechanism that will operate properly when needed (P=1 with perfect switching).

1. When all units are actively on-line having equal unit failure rates with k out of n required operational for success, the effective failure rate with repair is

$$\lambda_{eff\ k/n} = \frac{n!(\lambda)^{n-k+1}}{(k-1)!(\mu)^{n-k}}$$

and without repair is

$$\lambda_{eff\ k/n} = \frac{\lambda}{\sum_{i=k}^{n} \frac{1}{i}}$$

2. When two active on-line units operate with different failure with one of two required for success, the effective failure rate with repair is

$$\lambda_{eff\ \ 1/2} = \frac{\lambda_A \lambda_B [(\mu_A + \mu_B) + (\lambda_A + \lambda_B)]}{\mu_A \mu_B + (\mu_A + \mu_B)(\lambda_A + \lambda_B)}$$

and without repair is

$$\lambda_{eff\ \ 1/2} = \frac{\lambda_A^2 \lambda_B + \lambda_A \lambda_B^2}{\lambda_A^2 + \lambda_B^2 + \lambda_A \lambda_B}$$

3. When one standby off-line unit with n active on-line units required operating for success (with off-line spare assumed to have a failure rate of zero), and on-line units have equal failure rates, then the effective failure rate with repair is

$$\lambda_{eff\ \ n/n+1} = \frac{n[n\lambda + (1-P)\mu]\lambda}{\mu + n(P+1)\lambda}$$

and without repair is

$$\lambda_{eff\ n/n+1} = \frac{n\lambda}{P+1}$$

APPENDIX C

AVAILABILITY

The basic mathematical definition of availability is

$$\text{Availability} = A = \frac{\text{Up Time}}{\text{Total Time}} = \frac{\text{Up Time}}{\text{Up Time} + \text{Down Time}}$$

Actual assessment can involve substituting the time that comes from various forms of this basic equation. Thus, different combinations of elements combine to formulate different definitions of availability. Two different types of availability are described.

Inherent Availability (Ai)

Inherent availability is used when a system's availability is defined with respect only to operating time and corrective maintenance and is

$$Ai = \frac{MTBF}{MTBF + MTTR}$$

where MTTR is the mean time to repair an item and MTBF is the mean time between failure. Under this definition, other time periods are ignored such as standby, scheduled delay times, preventative maintenance, as well as administrative and logistic down time. Inherent availability is useful in determining basic system operational characteristics. However, it provides a very poor estimate of the true system's potential, because it provides no indication of the time required obtaining required field support.

Operational Availability (Ao)

Operational availability covers all segments of time that the equipment is intended to be operational. Here up time now includes operating time plus non-operating (standby) time (when equipment is assumed to be operable). Down time is expanded to include preventive and corrective maintenance and associated administrative and logistic lead-time. All are measured in clock time.

$$Ao = \frac{OT + ST}{OT + ST + TPM + TCM + ALDT}$$

Here OT is the operating time in use, ST is the standby time, TPM is the total preventive (scheduled) maintenance time, TCM is the total corrective (unscheduled) maintenance time, and ALDT is the administrative and logistic down time (delay-down time with no maintenance time). This definition is intended to be a realistic measure of equipment availability. A simpler common expression that is often used for Ao is

$$Ao = \frac{MTBM}{MTBM + MDT}$$

where MTBM is the mean time between maintenance actions and MDT is the mean down time.

Availability and Probability

Availability is essentially a probability number describing the probability to be available. From the way it is defined (up time/{up time + down time}), availability is a number between 0 and 1. Therefore, availability can be treated mathematically, similar to the way in which reliability R is described. For example, if the availability of subsystems A, B, and C is 0.9, 0.95, and 0.8, respectively, then the availability of these subsystems in series is (0.9)(0.95)(0.8) = 0.684. Thus, the serial system is available 68.4% of the time. Such treatment can be important in simplifying availability estimates.

APPENDIX REFERENCES

1. The Rome Laboratory Reliability Engineer's Toolkit, April 1993.