Beyond Miner’s Rule
Free Energy Damage Equivalence

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Miner's Rule - Energy Approach to Damage

- Miner's empirical rule was an important as it gave us the concept of damage

$$Damage = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_k}{N_k} = \sum_{i=1}^{K} \frac{n_i}{N_i}$$

- Today we can use an energy approach that goes beyond Miner's rule for it is more general and exact; and is reasonably practical and accurate approach at the measurable level.

$$Damage = \frac{\sum W_{actual}(t)}{W_{actual-failure}}$$

- *<u>The measurable work damage ratio:</u> consists of the actual work performed to the actual work needed to cause system failure.*

The Key Issue is the Denominator

- What is the amount of work to failure??

$$W_{actual-failure}$$

- If we know this we are in a good position to assess accumulative damage

- Is there a way to predict the work to failure based on a material property?

What Does Einstein's Equation Have to Do with this

- To understand this approach consider Einstein famous equation

$$E=mc^2$$

- This equation allows us to predict how much energy we can theoretically obtain from a given mass.

- We can ask, is there a classical analogy for assessing the potential useful work that can be achieved related to a known material property.

Material's Free Energy

- In thermodynamics, a materials free energy provides an assessment of the amount of useful work that a product can perform.
- This is not currently listed material property. Often too hard to calculate and is often treated for academic interest only.
- In reality, if we can asses a materials free energy for a particular type of work then it would be a useful property

Free Energy & Damage Equivalence

- Free energy is associated with the material useful work
- It is also equivalent to the amount of thermodynamic accumulated damage that can be allowed by a product.
- *The work that can be done on or by the system is then bounded by the system's free energy*

$$Work \leq \Delta \text{ Free Energy Change of the system}$$

- *ΔFree energy=0, the system is completely degraded*

Materials Maximum Work Strength For a Failure Mode

- In this paper we propose a materials Ultimate Work Energy (W_{UE}) for a given failure mode is the most measurable and useful property to assess a materials free energy, (analogous to Einstein's equation)

$$F_i - F_f = (\Delta FreeEnergy)_{Max} = W_{failure}(UE)$$

F_i =Initial free energy (before aging)
F_f =Final free energy (after aging)

Damage Equivalency To Free Energy

- ***Damage – Free energy equation***
- where P is the aging parameter of interest, C and K are constants, and t is time.

$$Damage = \frac{\Delta Free\ Energy}{(\Delta Free\ Energy)_{Max-damage}} = \frac{\Delta Free\ Energy}{W_{failure}(UE)},$$

$$and\ D = 1,\ when\ \Delta Free\ Energy = W_{failure}(UE)$$

Measurement Concept

- We can denote $W(UE)_{0+}$ as a measurement of the ultimate work energy for a very short time

$$W(UE)_{0+} \approx W(UE)$$

- The concept is to measure the ultimate work energy in a short time so that it is reasonably accurate and representative of the actual ultimate work energy.

Remaining Work

- Once we know the W(UE) for a particular failure mode, then energy can be subtracted when work is accomplished as damage accumulates.

$$Wr=W(UE)-Wi$$

Wr =Work remaining in a product
Wi =Interim work

- Damage D is

$$D=wi/W(UE)$$

Simple Example – Primary Battery

- Maximum work - Gibbs Free Energy, difficult to calculate

$$Max\ Work = -\ \Delta G$$

- 9V Battery has been measured, rated for 0.5 amp-hours

$$Max\ Work = 9v \times 0.5A \times 1hr\ (3600\ sec.) = 16{,}200\ joules$$

- We could measure this, 2 Ohm Resistor I=V/R=4.5 amps, $W(UE)_{0+}$= *measurement time is*

$$16{,}200\ J/(9V \times 4.5A) = 400\ seconds = 6.7\ Minutes$$

Simple Example – Primary Battery (Cont.)

- If the battery does work for ¼ of an hour at a rate of 0.1A, the energy used is

  $(Work)_i = 9V \times 0.1A \times \frac{1}{4}\ hr\ (900\ sec.) = 810\ Joules$

- Then the work remaining in the battery is

  $Wr = Wmax - Wi = 16{,}200 - 810 = 15{,}390\ Joules$

- $Damage = wi/Wue = 0.05$ or 5%

Fatigue And Ultimate Work Energy

- Fatigue life estimation is difficult for this approach, a function of size, material properties, metal treatment (such as annealed) surface condition etc
- The sine vibration cyclic work for G level of n cycles is found as

$$w = A\,G^{Y} n^{P}$$

- Consider N_1 cycles to fail at stress level G_1. Then damage at G_2 level for n_2 cycle is

$$Vibration\ Damage = \frac{w}{W_F} = \left(\frac{n_2}{N_1}\right)^{p}\left(\frac{G_2}{G_1}\right)^{Y}$$

Fatigue And Ultimate Work Energy (Cont.)

- When damage is 1, failure occurs
- This allows us to calculate the Acceleration Factor as

$$AF_D = \frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right) = \left(\frac{G_2}{G_1}\right)^b$$

- This is a commonly used for the acceleration factor in sinusoidal testing. For random vibration, substitute for G the random vibration Grms

Ultimate Work Energy - Stainless Steel Fatigue Life

- Fatigue is dominated by tensile force rather than compressive force
- Stainless steels ultimate tensile work energy is not available but could be calculate
- However, the ultimate tensile strength (stress units) is provided (a conjugate work dependent variable – work=stress x strain)

Properties	**Stainless 316L**
Yield strength	42 KSI (290 MPa)
Ultimate Tensile Strength	81 KSI (558 MPa)
Fatigue/endurance limit	39 Ksi (269 MPa)

Determining S-N Curve Example

- Experience has shown that for steel, the S-N curve ultimate strength is closer to 1000 Cycles for 90% of the ultimate strength.
- This is similar to finding the ultimate work energy at a reasonable amount of time on a battery; we might use 5 ohms instead of a short circuit.
- Furthermore it is well known that the endurance limit occurs around at 10^7 cycles.

Determining S-N Curve Example

- Therefore our two plot points for an S-N curve are

  $S_1 = 560 \times 0.9 = 504$ *MPa at* $N_1 = 1000$ *Cycles,*
  $S_2 = 309$ *MPa at* $N_2 = 10^7$ *cycles*

- Then from our equations we can write

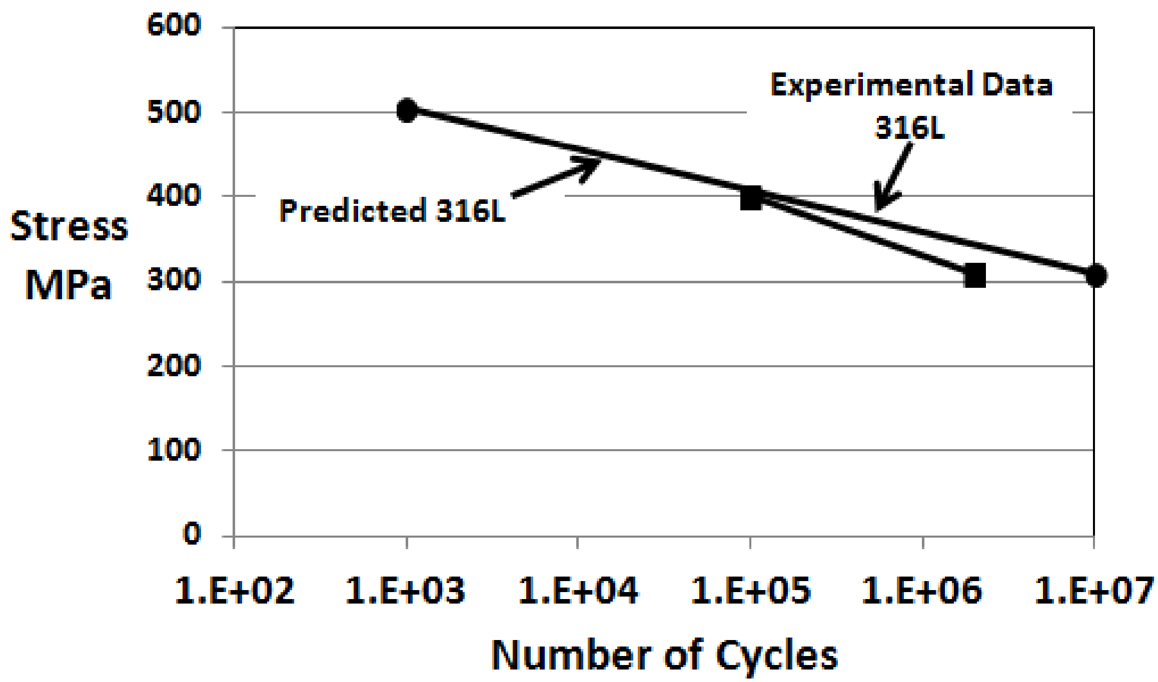
$$N_1 = N_2 \left(\frac{G_1}{G_2} \right)_{Sinusoidal}^{-b} \equiv N_2 \left(\frac{S_1}{S_2} \right)_{Sinusoidal}^{-b}$$

- where the slope is

  1/b=-(logS1-logS2)/(logN2-LogN1)= 18.8

Results

- Literature search comparison experiment to predicted shown below
- Comparison in the slope. The literature slope was 11.8.



Conclusions

- This paper goes beyond Miner's rule and we described a free energy approach to measuring damage
- Free energy – the useful work, has a maximum value that bound the work, we termed this the ultimate work energy that allows us to estimate the maximum allowed damage
- We anticipate some materials do not accumulate damage operated below a certain work strength degradation limit.