How Aging Laws Influence Parametric and Catastrophic Reliability Distributions

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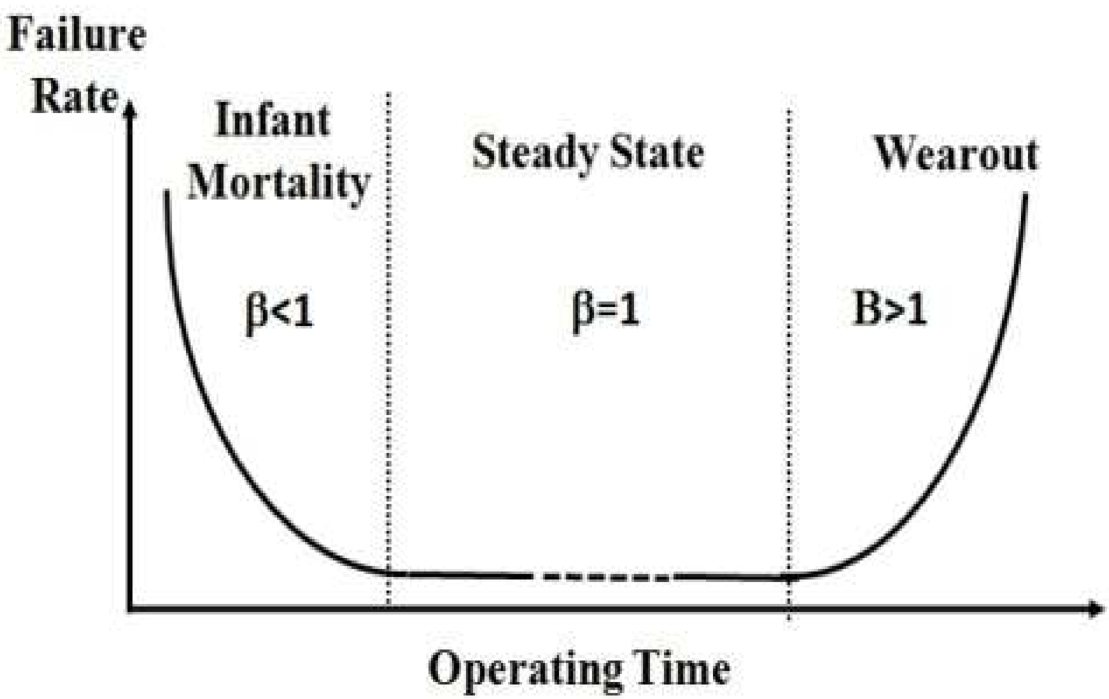
DfRSoft…

My Bio

- **Alec Feinberg** is the founder of DfRSoft. He has a Ph.D. in Physics and is the principal author of the books, Design for Reliability and Thermodynamic Degradation Science: Physics of Failure, Accelerated Testing, Fatigue, and Reliability Applications. DfRSoft provides consulting in reliability and shock and vibration, training classes and DfRSoftware. DfRSoftware, used by numerous companies, helps solve problems in HALT, accelerated test, Weibull analysis, physics of failure, predictions, reliability growth, quality, etc. and is also used to accelerate learning in his training classes. Alec has provided reliability engineering services in diverse industries (AT&T Bell Labs, Tyco Electronics, HP, NASA, etc) for over 35 years in aerospace, automotive and electrical and mechanical systems. He has provided training classes in Design for Reliability & Quality, Shock and Vibration-test, design, and assurance, HALT and ESD. Alec has presented numerous technical papers and won the 2003 RAMS best tutorial award for the topic, "Thermodynamic Reliability Engineering."

Reliability Distributions Fit the Bathtub Curve

- Most common example is Weibull power law exponent Beta



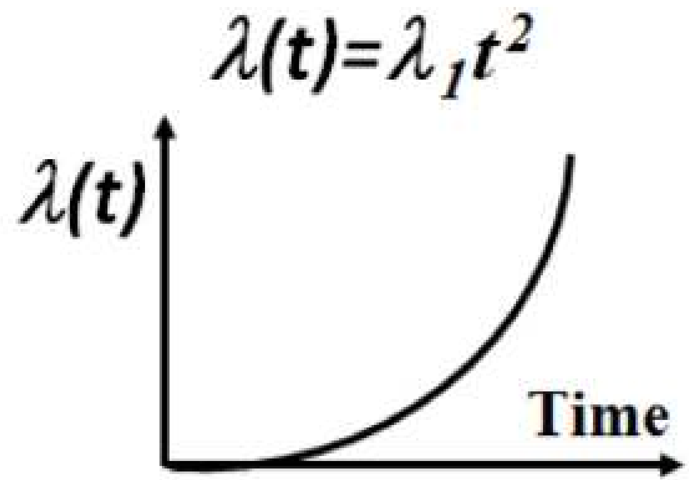
Example: Weibull Beta Fit on Wear Out

- If we tried to fit the wear out area, what model would we use? A basic power law expression is the Weibull choice

- When we simplify the complex looking Weibull failure rate, it is just a simple Power Law form

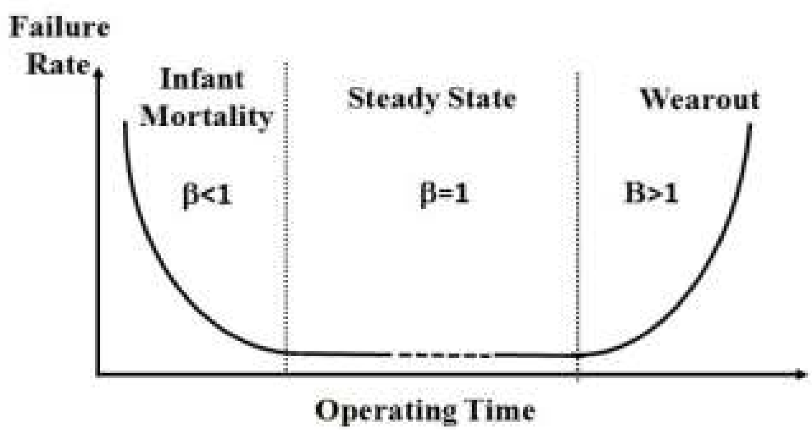
$$\lambda(t) = \frac{\beta}{\alpha^{\beta}}(t)^{\beta-1} \equiv \lambda_1 t^{\phi}$$

Where $\beta=3$, $\phi=2$, $\frac{\beta}{\alpha^{\beta}} \equiv \lambda_1$



Bathtub Curve and Physics of Failure

- Is there a connection between the bathtub curve and physics of failure aging laws?
- If so, then we can support the conclusion that parametric aging effects the underlying failure rate distribution



Common aging laws

$$\begin{cases} P(t) = Ct^K \\ \boldsymbol{P = A \log(1 + b\, t)} \\ R = R_o\, e^{-E_a / K_B T} \end{cases}$$

Most Popular Aging Laws In The Literature

- ***Power Law:*** $$P(t) = Ct^K$$
- where P is the aging parameter of interest, C and K are constants, and t is time. Example, most accelerated testing equations (Coffin-Manson, Vibration deg rate, Creep stages, thin films resistors, capacitor voltage life, etc.
- ***Log time Aging*: *P=A log(1+b t)***
  - where A, and b are constants, P is the parameter of interest and t is time. *Example: Transistor gate drift, crystal frequency drift*, primary battery degradation, primary stage 1 creep…
- ***Arrhenius Activation:*** $$R = R_o e^{-E_a / K_B T}$$
  - where R is the aging rate of a parameter, R_o is a constant related to a time constant $1/t_o$. The rate R goes inversely with time. Used to model most reliability temperature failure rate occurances…

Bathtub Statistics Works But What About the Underlying Physics?

- True statisticians will say whatever fits the data best, use that distribution…
- However, Let's ask a deeper question, why do we see this type of failure rate shape??
- Is there perhaps a reason to select one distribution compared to another due to the physics?
- What about the underlying aging law?
- Are the distribution parameters related to the aging law??

Two Types of Reliability Failure Rates

- Parametric Failure
  - Typically graceful degradation, and often predictable
- Catastrophic Failure:
  - Sudden failure
  - Can result from underlying parametric aging law
- Parametric degradation can be associated with catastrophic failure
  - Example 20% drift unacceptable –a failure limit, or 80% defects create a catastrophic sudden crack & failure

Log(time) Aging Laws and the Lognormal Distribution

Example of Parametric Failure Rate Connection to Lognormal aging in time

- *Let's first illustrate that if parts are normally distributed and age in log-time, then their failure rate is lognormal.*

age in log-time

$$P = A\,\log(1 + b\,t)$$

→

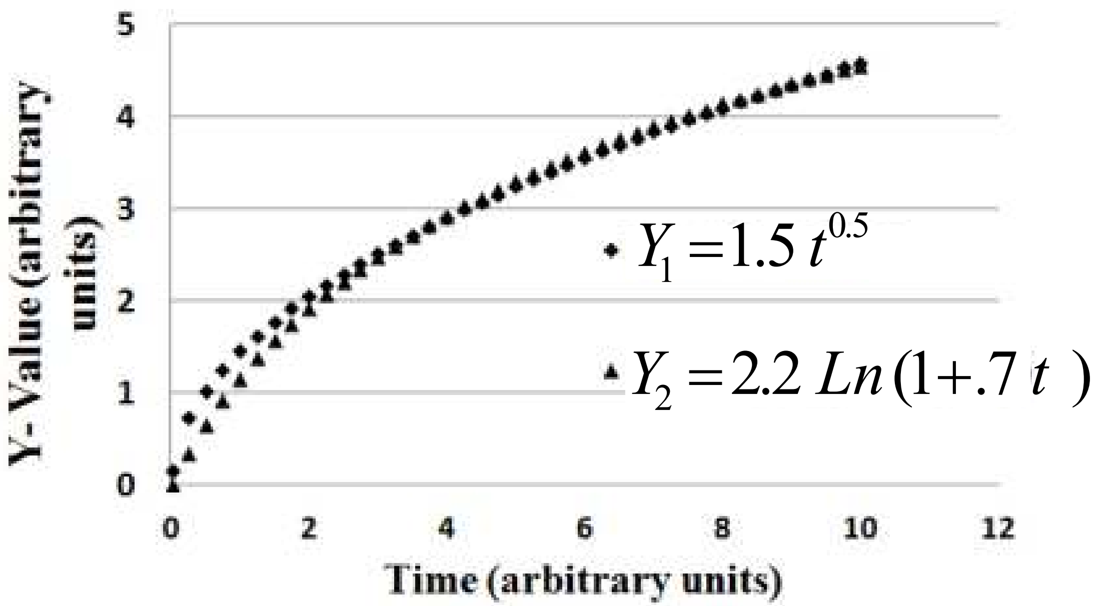
Lognormal PDF

$$f(t) = \frac{1}{\sigma t \sqrt{2\pi}} \exp\left\{ -\frac{1}{2}\left(\frac{\ln t - \ln t_{50}}{\sigma}\right)^2 \right\}$$

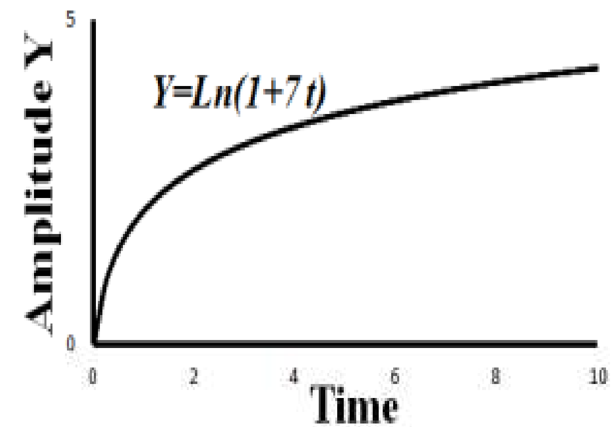
What Does Log(time) Drift Look Like?

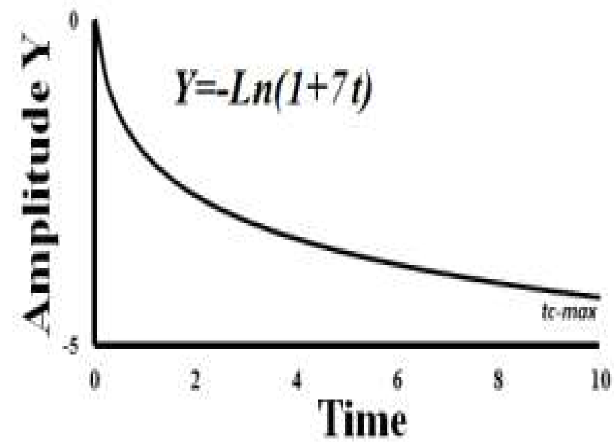
- Log(time drift) also looks like power law aging for 0<K<1

$$Y_2 = 2.2\,Ln(1+.7\,t\,) \leftrightarrow Y = Ct^K$$



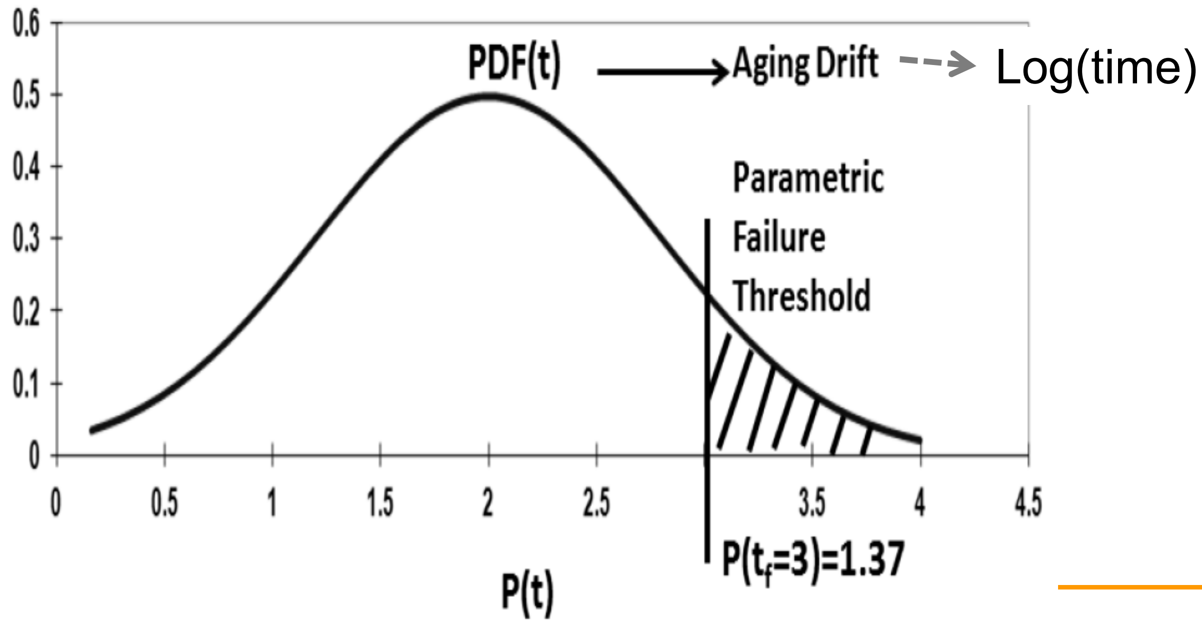
Log(time drift)





Visualization for Normal Distribution aging to the right

- Here the parameter P(t) passes the parametric failure threshold over time, if is does so due to log time drift then we would like to show that the failure rate will be lognormal failure rate in time.



Log time aging simplified

- To that end, the general form of the a log time model is (see references)

$$P = A\ln(1+bt) \approx Ct^K \qquad \text{where } 0 < K < 1$$

- and to simplify, when $bt >> 1$ we can write

$$P \approx A\ln(bt)$$

- Here P is the parameter of interest, such as creep{$P=\Delta\varepsilon$=Strain), beta gain of a transistor's aging, or perhaps crystal frequency drift and so forth.

Inserting Log time aging into normal PDF

- When manufactured parts are normally distributed, a parameter of interest can be statistically assessed using Gaussian probability density function (pdf) *g(p, t)*

$$g(p,t)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{p(t)-\bar{p}(t)}{\sigma}\right)^2\right]$$

- Here *P* is the parameter of interest like creep. Now consider that the parameter is aging according to a log-time equation then by substitution

$$g(\ln t:t_{50},\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln t-\ln t_{50}}{\sigma}\right)^2\right]$$

Where for purposes of illustration in Equation 4 we have let A=b=1

Obtaining the Lognormal distribution form log time aging

- It is customary to change variables so that we may formally obtain the lognormal distribution for the above equation, this occurs typically when integrating for the CDF i.e.,

$$g(\ln t)d\ln t = g(\ln t)\frac{d\ln t}{dt}dt = g(\ln t)\frac{dt}{t}$$

- Although used in integration for the CDF it is valid to now write the pdf in its desired form

$$f(t:t_{50},\sigma) = \frac{1}{\sigma t\sqrt{2\pi}} \quad \exp\left[-\frac{1}{2}\left(\frac{\ln t - \ln t_{50}}{\sigma}\right)^2\right]$$

- Here, the function $f(t{:}t_{50}, s)$ is the lognormal probability density function as desired yielding lognormal failure rate

Lognormal Distribution Model

- **Probability Density Function**

$$f(t) = \frac{1}{\sigma t \sqrt{2\pi}} \exp\left\{ -\frac{1}{2} \left(\frac{\ln t - \ln t_{50}}{\sigma} \right)^2 \right\}$$

- **Cumulative Distribution Function**

$$F(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^t \frac{dx}{x} \exp - \frac{1}{2} \left(\frac{\ln(x / x_{50})}{\sigma} \right)^2 \qquad = \frac{1}{2}\left[1 + erf\left(\frac{\ln(t / t_{50})}{\sqrt{2}\sigma} \right) \right]$$

- **Failure Rate**

$$\lambda(t) = \frac{f(t)}{1 - F(t)}$$

- **Median = t_{50}**
- **Shape Parameter**

$$\sigma = \ln(t_{50} / t_{16})$$

Physical Implications Related to Log-time Aging

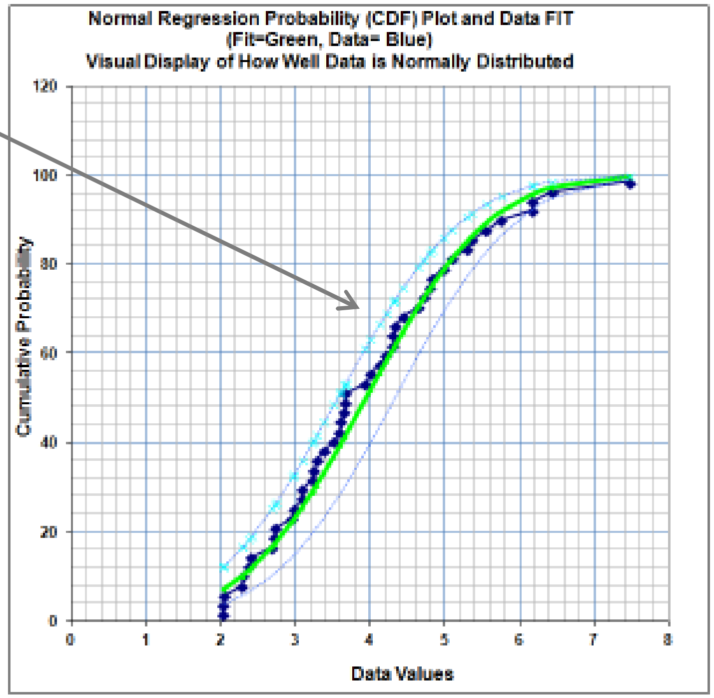
- ***When a manufactured part has a key parameter that is distributed normally, and ages in log-time, its failure rate is generally lognormally distributed. This could also be the case for power law aging models that are typically found empirically as well when the power 0<K<1.***

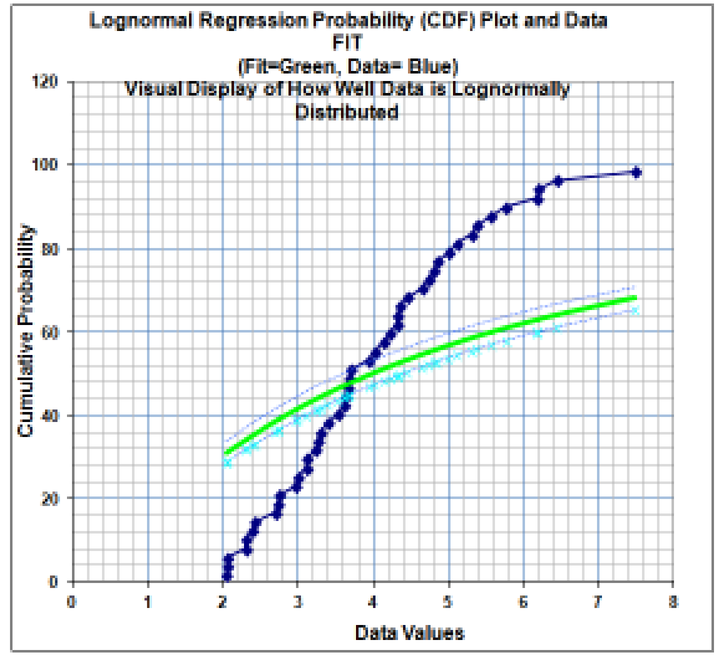
- *Although we have described this for parametric failure, it can apply to catastrophic failure. For example, if a transistor is aging most of its lifetime in log-time then suddenly fails catastrophically, but it was due to the underling log-time aging mechanism like gate leakage, then the transistor's failure distribution is likely lognormal.*

Example Normally Distributed a-Values

	a
1	2.04
2	2.04
3	2.05
4	2.29
5	2.30
6	2.38
7	2.41
8	2.69
9	2.73
10	2.74
11	2.96
12	2.98
13	3.10
14	3.10
15	3.23
16	3.25
17	3.29
18	3.39
19	3.52
20	3.60
21	3.61
22	3.66
23	3.67
24	3.68
25	3.94
26	4.01
27	4.13
28	4.22
29	4.31
30	4.32
31	4.34
32	4.44
33	4.65
34	4.72
35	4.80
36	4.84
37	4.99
38	5.10
39	5.30
40	5.37
41	5.56
42	5.76
43	6.17
44	6.18
45	6.43
46	7.48

Data Normality Analysis Linear Regression of Data to F(x) CDF			
Enter Confidence	Confidence Assessment		
	Sigma	Mean	Rho
95			Normality Test
Observation	1.295746374	3.951906378	0.989873518

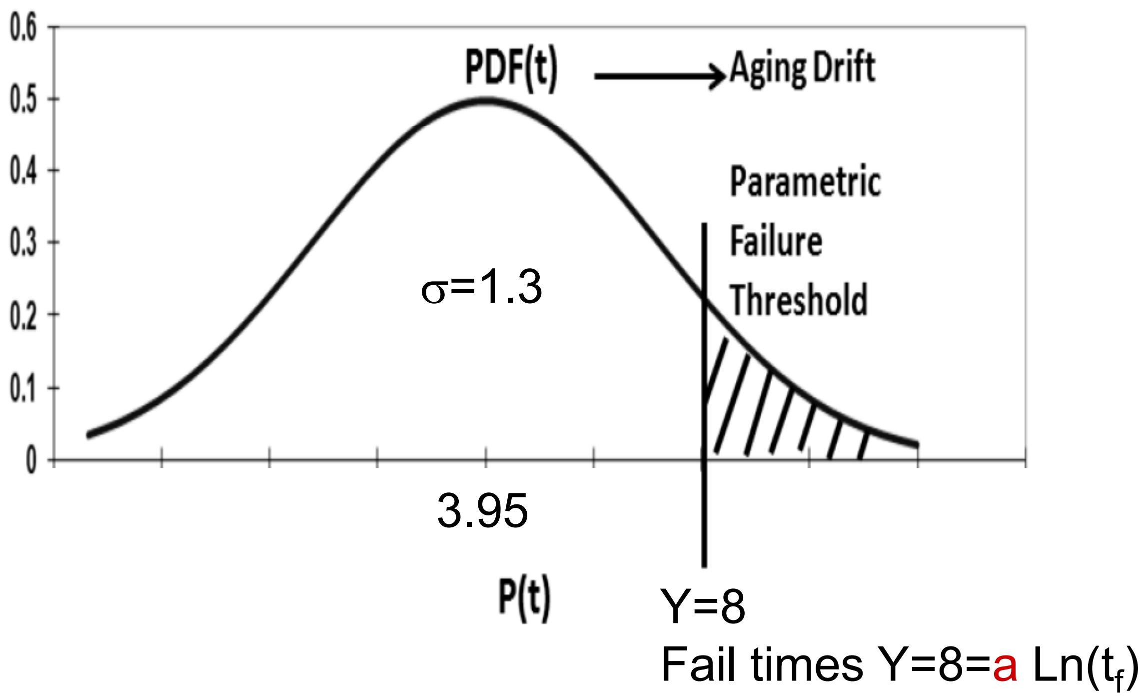




- Example of normally distributed values. Note, lognormal poor fit

Distribution Aged in Log(time)

- Select a failure criteria Y=8, values age according to equation Y=a $Ln(t_f)$



Example (Lognormal Aging Fit)

$$Y = a \times Ln(t_f)$$

- Normally distributed values log(time) aged to value $t_f = 8$ failure threshold time
- Examples: first value

$Y = 2.04 \times Ln(50.75) = 8$

Last value

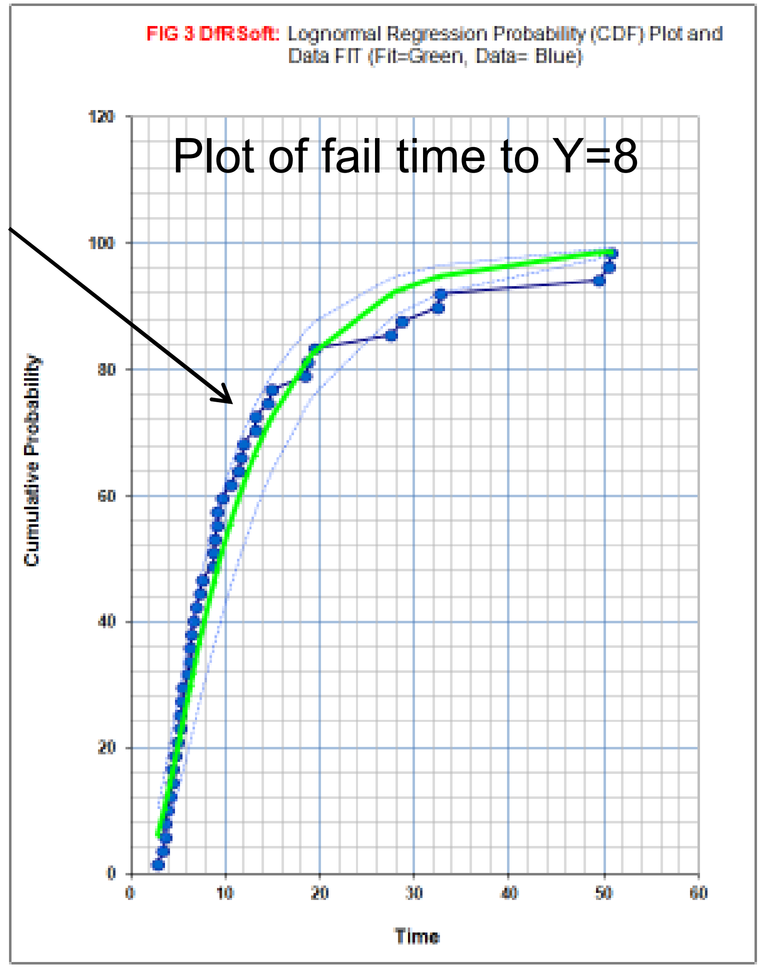
$Y = 7.48 \times Ln(2.92) = 8$

We see good fit to t_f fail times are lognormal distributed Rho = 95%

	a	Fail times for Y=8
1	2.04	50.75
2	2.04	50.56
3	2.05	49.49
4	2.29	32.80
5	2.30	32.42
6	2.38	28.74
7	2.41	27.50
8	2.69	19.55
9	2.73	18.79
10	2.74	18.51
11	2.96	14.91
12	2.98	14.61
13	3.10	13.21
14	3.10	13.21
15	3.23	11.92
16	3.25	11.72
17	3.29	11.37
18	3.39	10.60
19	3.52	9.72
20	3.60	9.23
21	3.61	9.16
22	3.66	8.90
23	3.67	8.86
24	3.68	8.80
25	3.94	7.63
26	4.01	7.35
27	4.13	6.92
28	4.22	6.66
29	4.31	6.39
30	4.32	6.38
31	4.34	6.32
32	4.44	6.05
33	4.65	5.59
34	4.72	5.44
35	4.80	5.29
36	4.84	5.23
37	4.99	4.97
38	5.10	4.79
39	5.30	4.52
40	5.37	4.44
41	5.56	4.21
42	5.76	4.01
43	6.17	3.66
44	6.18	3.65
45	6.43	3.47
46	7.48	2.92

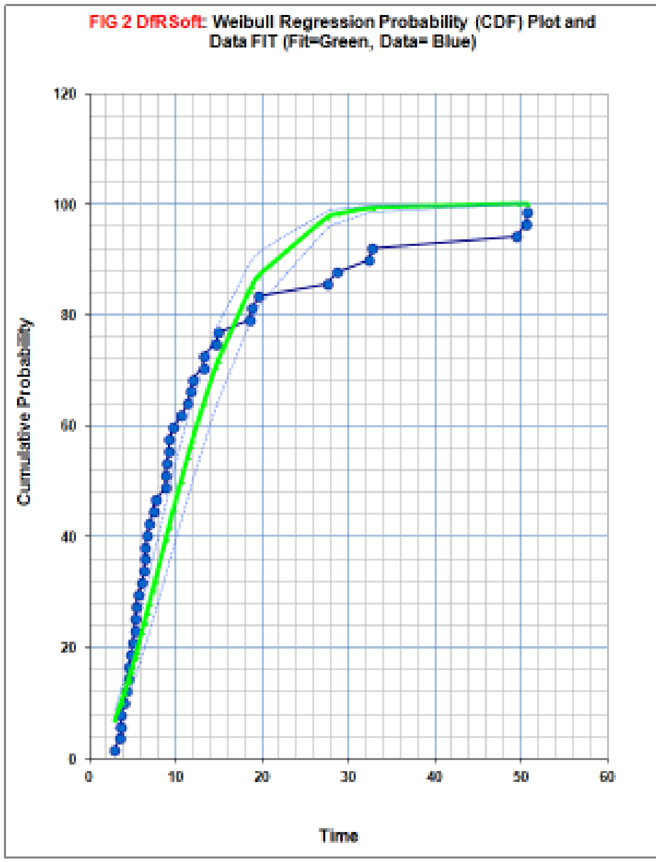
Good Fit

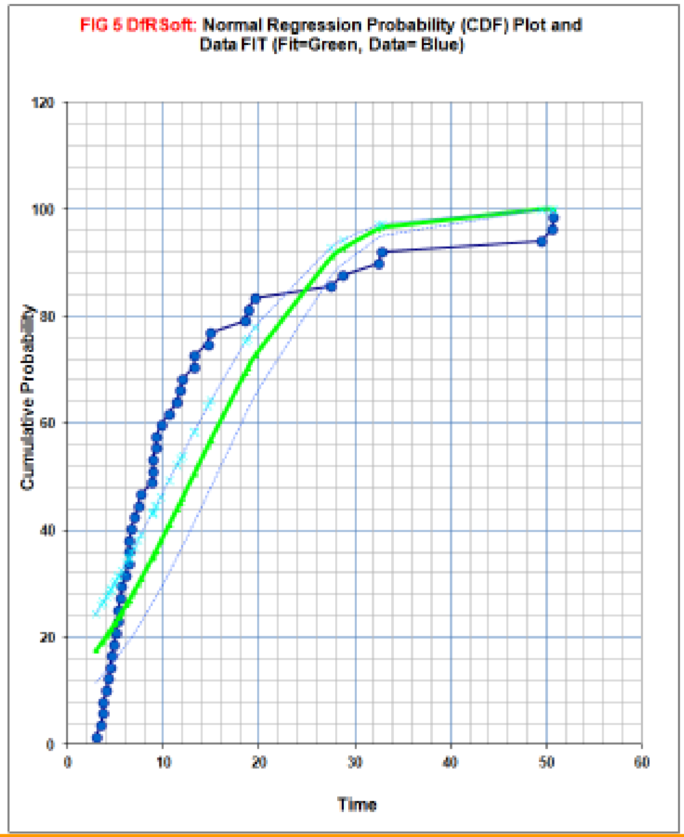
Confidence	Lognormal Regression Results			
Enter %	Sigma	Median Life	Mean Life	Rho
80	Shape Param	(50% failure)	(Average)	
RRX	0.760644513	9.506802604	12.69613534	0.947969914



Note - Lognormal is best fit compared to:

- Weibull Rho 83% Normal Rho 72%



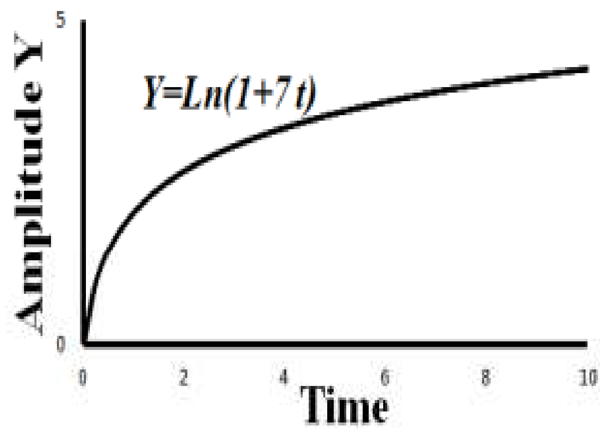


Thermally Activated Time-dependent (TAT)

- Origin of log time aging model can be thermally activated processes. The author has published numerous publications on this and these are cited (see References below) often used for transistor aging assessment

$$a = \frac{\Delta P}{P} \cong A \ln[1 + B t]$$

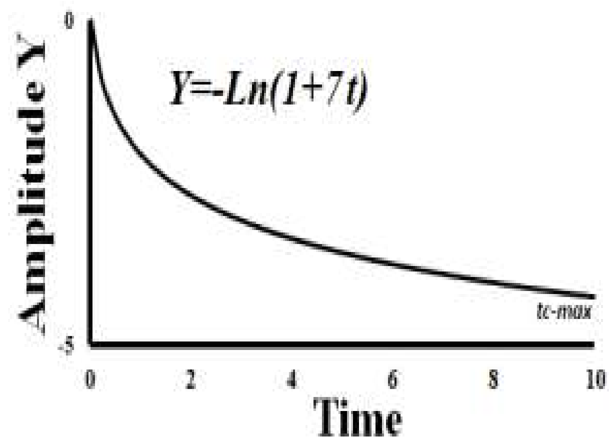
$$A = \frac{K_B T}{y_1} \quad and \quad B = \frac{\nu(T) y_1}{K_B T}$$



- Origin of TAT model – Thermally

Activated Processes, y1 above related to Free energy ϕ

$$\frac{dp}{dt} = \nu \quad \exp\left(- \frac{\phi}{K_B T} \right)$$



Reference: Ch. 6 & 7 of my PoF book see last pages, Good application is for assessing key aging of transistor device parameters also in numerous publications on this. Ch 7 has application of this TAT model.

Power Aging Laws and the Weibull Distribution

Example of Parametric Failure Rate Connection to Weibull

- *Next we would like to illustrate that if parametric age occurs nonlinear in time (power law) then we can associate the power law with the Weibull failure rate parameters at least in the parametric case.*

Power Law aging

$$P(t) = Ct^{K}$$

→

Weibull Failure Rate

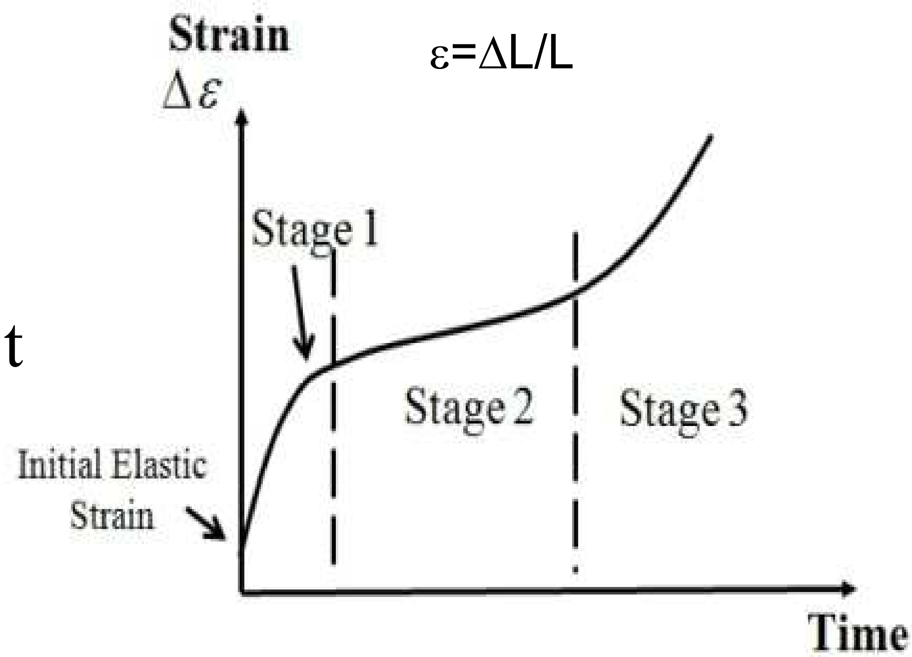
$$\lambda(t) = \frac{\beta}{\alpha^{\beta}}(t)^{\beta-1}$$

Aging Power Laws and the Weibull Distribution – Influence on Beta

- Many parametric aging laws have power law dependence. Consider a very simplified creep model

$$\Delta\varepsilon = \varepsilon_o t^N \qquad \begin{cases} N < 1 & Stage\,1 \\ N \geq 1 & Stage\,2 \\ N > 1 & Stage\,3 \end{cases}$$

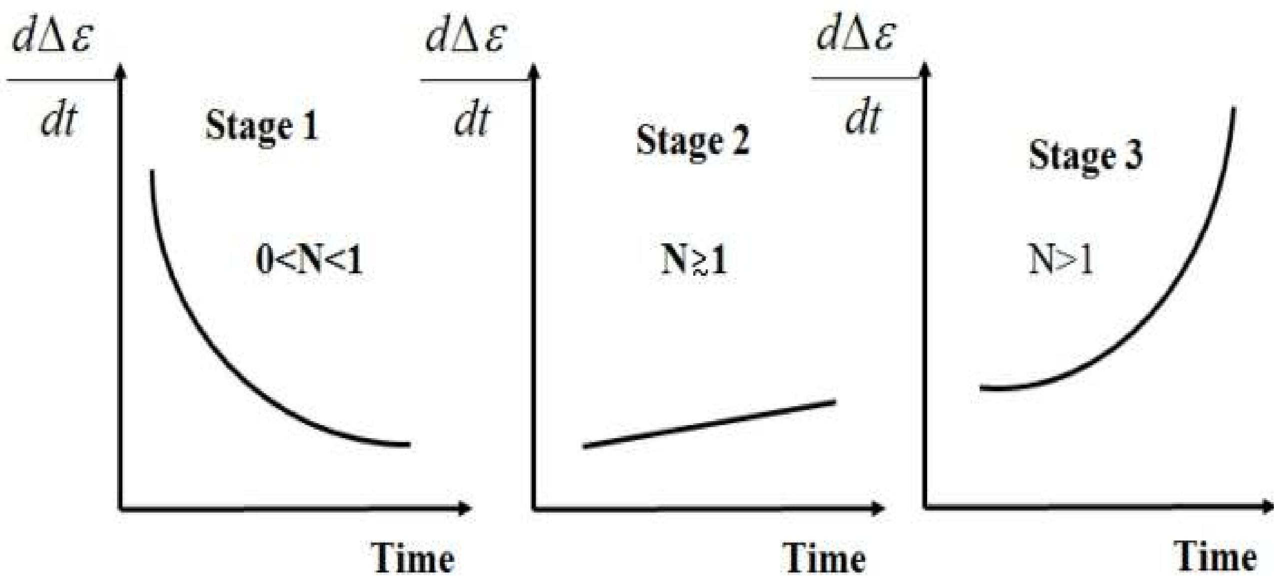
- Were Δε is the creep strain and t is the time, and ε_o and N are constants of the creep model phases (note: ε_o has units of strain/time and strain itself is unitless so really 1/time)



Creep Rate Stages have Similar Form to Bathtub Curve

- The creep rate is $$\frac{d\Delta\varepsilon}{dt} = \varepsilon_o N t^{N-1}$$

Similar shape to Infant Mortality	Similar shape to Steady State	Similar shape to Wear Out



Inferences

$$\lambda(t) = \frac{\beta}{\alpha^{\beta}} (t)^{\beta-1}$$

Creep Rate, Weibull Fail Rate Comparison

$$N = \beta$$

$$\varepsilon_o^{*} = \frac{1}{\alpha^{\beta}}$$

* Ballpark value see g(E) other slides

Parametric Creep Rate, Weibull Failure Rate & Bathtub Curve

- A direct comparisons between creep rate and the failure rate,
- Results N~ β.
- Here N between 0 and 1, say β=N=1/2, indicating that creep is in the Creep Primary Stage 1, similar to infant mortality region (early failure)
- If N=1, constant creep rate, Secondary Creep Stage 2, similar to steady state region of the bathtub curve as $\beta=1$.
- N=$\beta>1$, Tertiary Creep Stage 3 similar to wearout phase of the bathtub curve.

Manipulation of failure rate and creep failure

- The expected fraction of devices that will fail ΔE in the time interval *Δt* then is $\overline{\lambda(t)} = \frac{\Delta E}{\Delta t}$ and

$$\lambda(t) = \lim_{\Delta \to 0} \frac{\Delta E}{\Delta t} = \frac{dE}{dt}$$

$$= -\frac{1}{R(t)} \frac{dR(t)}{dt} \quad \text{where} \quad \Delta E = \frac{R(t) - R(t + \Delta t)}{R(t)}$$

- Parametric failure threshold $t_{failure}$

$$\Delta \varepsilon_{failure}(t) = \varepsilon_o \; t^N_{failure}$$

Basic Proof for Parametric Failure Rate

- The expected fraction of devices that will fail $\Delta E(\Delta\varepsilon(t))$ in the time interval Δt then must be a function of the aging law so that the failure rate is

$$\lambda(t) = \frac{dE}{d(\Delta\varepsilon_f(t))}\frac{d(\Delta\varepsilon_f(t))}{dt} = g(E)\,\bar{\varepsilon}_o N\, t_f^{N-1}$$

Recall

$$\lambda(t) = \frac{\beta}{\alpha^\beta}(t)^{\beta-1}$$

- By comparison to Weibull failure rate we have

$$N = \beta \quad \text{and} \quad g(E)\overline{\varepsilon_0} = (1/\alpha)^\beta \quad or \quad \overline{\varepsilon_0} = k(1/\alpha)^\beta$$

- where $$g(E) = \frac{dE}{d(\Delta\varepsilon_{failure}(t))} = 1/k$$

See example for k

g(E) is dependent on the parametric failure threshold value as is Weibull Alpha

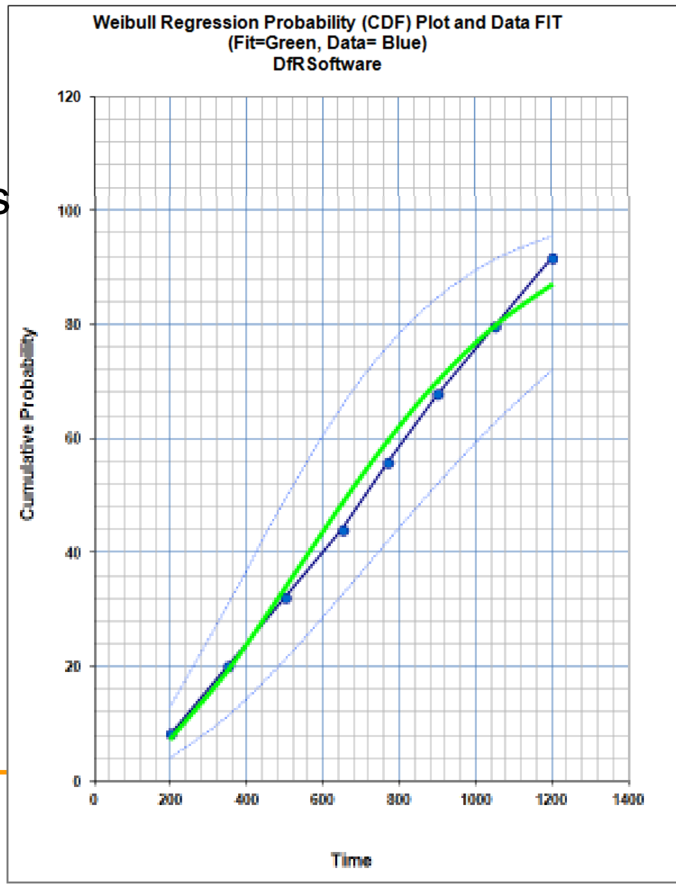
Example: Weibull Analysis and PoF Law

- Weibull Analysis

Confidence	Weibull Regression R		
Enter Percent	Beta	Charac Life	MTBF
80		(63.2% failure)	Expectation
RRX	1.818488305	812.8885144	722.5453664

Weibull Failure Times

200
350
500
650
770
900
1050
1200



$$\Delta\varepsilon_{failure}(t) = \varepsilon_o \; t_{failure}^{N}$$

Failure Criteria=10% creep elongation

N=1.8=β

ε_o	$t_{failure}$	$\Delta\varepsilon_{failure}$
7.21E-06	200	0.10
2.63E-06	350	0.10
1.39E-06	500	0.10
8.64E-07	650	0.10
6.37E-07	770	0.10
4.81E-07	900	0.10
3.65E-07	1050	0.10
2.87E-07	1200	0.10

$$\Delta\varepsilon_{failure} = 0.000000287x(1200)^{1.8} = 0.1$$

Characteristic Life and Average value of ε_o

- Average ε_o is related to characteristic life

$$\overline{\varepsilon_o} = k\frac{1}{\alpha^{\beta}} = k\frac{1}{813^{1.8}} = k5.8E-6$$

$$\overline{\varepsilon_o} = 1.7E-6$$

From data prev. page

k=1.7/5.8=0.293=1/g(E)

Recall $g(E)\overline{\varepsilon_0} = (1/\alpha)^{\beta}$

We see ballpark

$$\overline{\varepsilon_o} \sim (1/\alpha)^{\beta}$$

and g(E)=3.4

Here are Some Values You Can Check and note…

- Playing around I got

$\Delta\varepsilon_{failure}$	α	**g(E)**
10%	813	3.4
50%	1987	0.68
100%	2921	0.34

- So g(E)=1 when $\overline{\varepsilon_0} = (1/\alpha)^{\beta}$

solving this occurs when α=1604 hours which looks like it will occur somewhere between 10 and 50% $\Delta\varepsilon_{failure}$

- As we noted g(E) is dependent on this parametric failure criteria

Conclusions

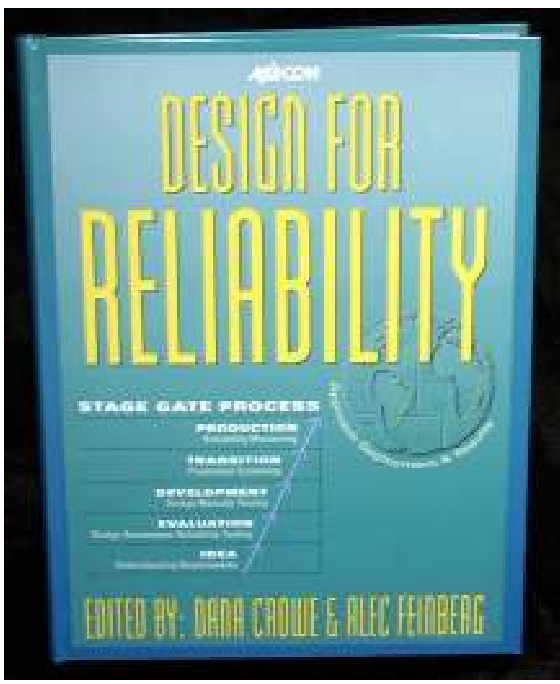
- *We have now connected both the Weibull Model and the Lognormal model to physical aging laws.*
- *This analysis is for Parametric aging laws. However, when parametric aging dominates a catastrophic failure event, it appears likely that the catastrophic failure rate is related to the parametric aging law.*
- *Alternately, we have also connected distributions to parametric aging laws. This means we can often deduce the physics of aging from failure distribution parameters, like Weibull beta.*
- ***Appropriate references for citing this work:***
  - A. Feinberg, "How Aging Laws Influence Parametric and Catastrophic Reliability Distributions," *RAMS 2017 Conference, and in* ieeexplore.ieee.org/iel7/7879516/7889646/07889800.pd,
  - Also (next page)

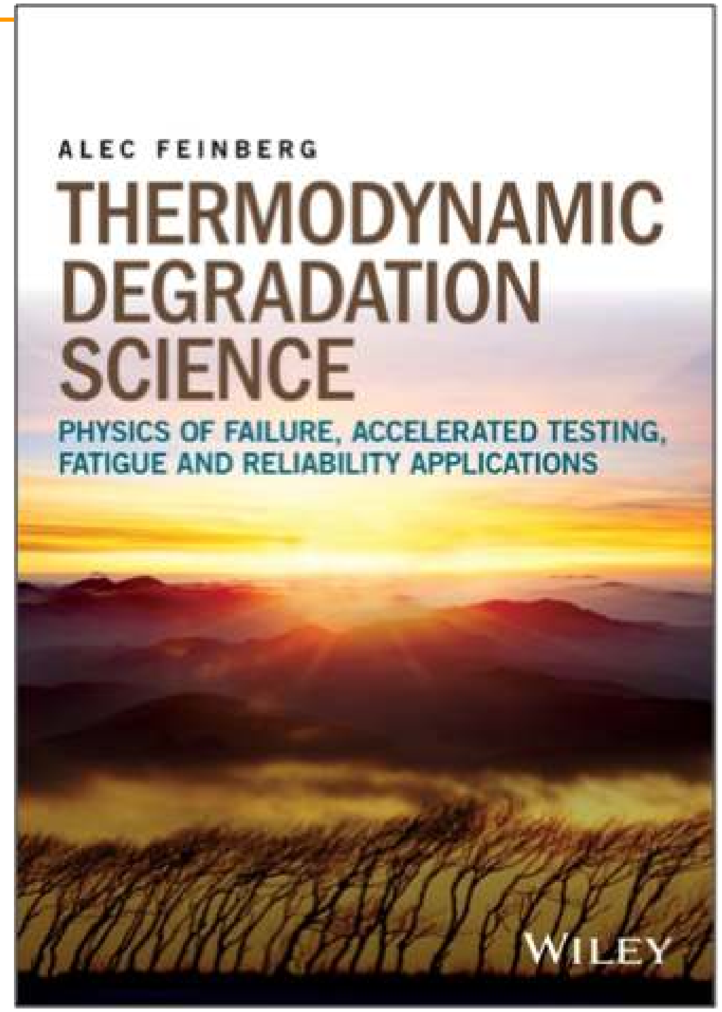
References (cont.):

- The original material is in Chapter 9 of my book.
- New methods for physics of failure and accelerated testing. →

Also see Software DfRSoft.com

My Other Book (CRC Press)





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