
Beyond Miner's Rule

Free Energy Damage Equivalence

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Miner's Rule - Energy Approach to Damage

- Miner's empirical rule was an important as it gave us the concept of damage

$$Damage = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots + \frac{n_k}{N_k} = \sum_{i=1}^K \frac{n_i}{N_i}$$

- Today we can use an energy approach that goes beyond Miner's rule for it is more general and exact; and is reasonably practical and accurate approach at the measurable level.

$$Damage = \frac{\sum W_{actual}(t)}{W_{actual-failure}}$$

- The measurable work damage ratio: consists of the actual work performed to the actual work needed to cause system failure.



The Key Issue is the Denominator

- What is the amount of work to failure??

$$W_{\text{actual-failure}}$$

- If we know this we are in a good position to assess accumulative damage
- Is there a way to predict the work to failure based on a material property?



What Does Einstein's Equation Have to Do with this

- To understand this approach consider Einstein famous equation

$$E=mc^2$$

- This equation allows us to predict how much energy we can theoretically obtain from a given mass.
- We can ask, is there a classical analogy for assessing the potential useful work that can be achieved related to a known material property.



Material's Free Energy

- In thermodynamics, a materials free energy provides an assessment of the amount of useful work that a product can perform.
- This is not currently listed material property. Often too hard to calculate and is often treated for academic interest only.
- In reality, if we can assess a materials free energy for a particular type of work then it would be a useful property



Free Energy & Damage Equivalence

- Free energy is associated with the material useful work
- It is also equivalent to the amount of thermodynamic accumulated damage that can be allowed by a product.
- *The work that can be done on or by the system is then bounded by the system's free energy*

Work \leq Δ Free Energy Change of the system

- *Δ Free energy=0, the system is completely degraded*



Materials Maximum Work Strength For a Failure Mode

- In this paper we propose a materials Ultimate Work Energy (W_{UE}) for a given failure mode is the most measurable and useful property to assess a materials free energy, (analogous to Einstein's equation)

$$F_i - F_f = (\Delta Free Energy)_{Max} = W_{failure}(UE)$$

F_i =Initial free energy (before aging)

F_f =Final free energy (after aging)



Damage Equivalency To Free Energy

- *Damage – Free energy equation*
- where P is the aging parameter of interest, C and K are constants, and t is time.

$$Damage = \frac{\Delta Free Energy}{(\Delta Free Energy)_{Max-damage}} = \frac{\Delta Free Energy}{W_{failure}(UE)},$$

and $D = 1$, when $\Delta Free Energy = W_{failure}(UE)$



Measurement Concept

- We can denote $W(UE)_{0+}$ as a measurement of the ultimate work energy for a very short time

$$W(UE)_{0+} \approx W(UE)$$

- The concept is to measure the ultimate work energy in a short time so that it is reasonably accurate and representative of the actual ultimate work energy.



Remaining Work

- Once we know the $W(UE)$ for a particular failure mode, then energy can be subtracted when work is accomplished as damage accumulates.

$$W_r = W(UE) - W_i$$

W_r = Work remaining in a product

W_i = Interim work

- Damage D is

$$D = w_i / W(UE)$$



Simple Example – Primary Battery

- Maximum work - Gibbs Free Energy, difficult to calculate

$$\text{Max Work} = -\Delta G$$

- 9V Battery has been measured, rated for 0.5 amp-hours

$$\text{Max Work} = 9\text{v} \times 0.5\text{A} \times 1\text{hr} (3600 \text{ sec.}) = 16,200 \text{ joules}$$

- We could measure this, 2 Ohm Resistor $I = V/R = 4.5$ amps, $W(UE)_{0+} = \text{measurement time is}$

$$16,200 \text{ J} / (9\text{V} \times 4.5\text{A}) = 400 \text{ seconds} = 6.7 \text{ Minutes}$$



Simple Example – Primary Battery (Cont.)

- If the battery does work for $\frac{1}{4}$ of an hour at a rate of 0.1A, the energy used is

$$(Work)_i = 9V \times 0.1A \times \frac{1}{4} hr (900 sec.) = 810 \text{ Joules}$$

- Then the work remaining in the battery is

$$W_r = W_{max} - W_i = 16,200 - 810 = 15,390 \text{ Joules}$$

- $Damage = w_i / W_{ue} = 0.05$ or 5%



Fatigue And Ultimate Work Energy

- Fatigue life estimation is difficult for this approach, a function of size, material properties, metal treatment (such as annealed) surface condition etc
- The sine vibration cyclic work for G level of n cycles is found as

$$w = A G^Y n^P$$

- Consider N_1 cycles to fail at stress level G_1 . Then damage at G_2 level for n_2 cycle is

$$\text{Vibration Damage} = \frac{w}{W_F} = \left(\frac{n_2}{N_1} \right)^P \left(\frac{G_2}{G_1} \right)^Y$$



Fatigue And Ultimate Work Energy (Cont.)

- When damage is 1, failure occurs
- This allows us to calculate the Acceleration Factor as

$$AF_D = \frac{T_1}{T_2} = \left(\frac{N_1}{N_2} \right) = \left(\frac{G_2}{G_1} \right)^b$$

- This is a commonly used for the acceleration factor in sinusoidal testing. For random vibration, substitute for G the random vibration Grms



Ultimate Work Energy - Stainless Steel

Fatigue Life

- Fatigue is dominated by tensile force rather than compressive force
- Stainless steels ultimate tensile work energy is not available but could be calculate
- However, the ultimate tensile strength (stress units) is provided (a conjugate work dependent variable – $\text{work} = \text{stress} \times \text{strain}$)

Properties	Stainless 316L
Yield strength	42 KSI (290 MPa)
Ultimate Tensile Strength	81 KSI (558 MPa)
Fatigue/endurance limit	39 Ksi (269 MPa)



Determining S-N Curve Example

- Experience has shown that for steel, the S-N curve ultimate strength is closer to 1000 Cycles for 90% of the ultimate strength.
- This is similar to finding the ultimate work energy at a reasonable amount of time on a battery; we might use 5 ohms instead of a short circuit.
- Furthermore it is well known that the endurance limit occurs around at 10^7 cycles.



Determining S-N Curve Example

- Therefore our two plot points for an S-N curve are

$S_1 = 560 \times 0.9 = 504 \text{ MPa}$ at $N_1 = 1000 \text{ Cycles}$,

$S_2 = 309 \text{ MPa}$ at $N_2 = 10^7 \text{ cycles}$

- Then from our equations we can write

$$N_1 = N_2 \left(\frac{G_1}{G_2} \right)_{\text{Sinusoidal}}^{-b} \equiv N_2 \left(\frac{S_1}{S_2} \right)_{\text{Sinusoidal}}^{-b}$$

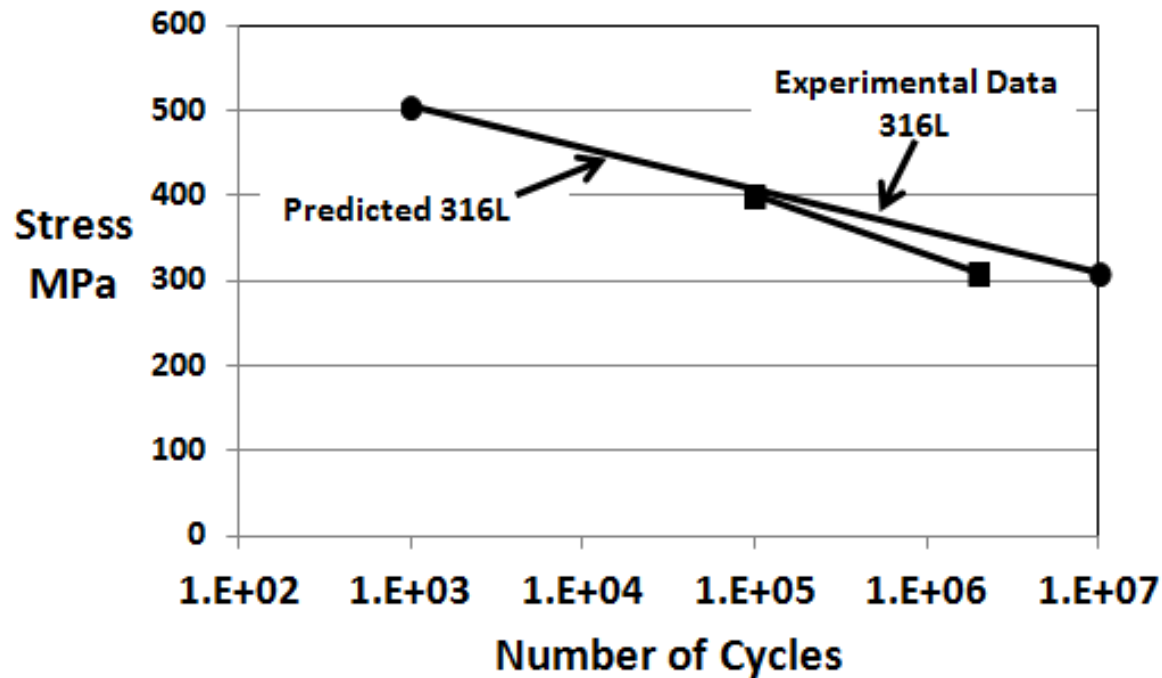
- where the slope is

$$1/b = -(\log S_1 - \log S_2) / (\log N_2 - \log N_1) = 18.8$$



Results

- Literature search comparison experiment to predicted shown below
- Comparison in the slope. The literature slope was 11.8.



Conclusions

- This paper goes beyond Miner's rule and we described a free energy approach to measuring damage
- Free energy – the useful work, has a maximum value that bound the work, we termed this the ultimate work energy that allows us to estimate the maximum allowed damage
- We anticipate some materials do not accumulate damage operated below a certain work strength degradation limit.

